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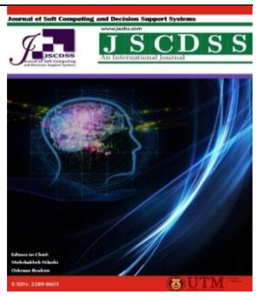

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A Comparative Study of CCR-(ε-SVR) and CCR-(ν-SVR) Models for Efficiency Prediction of Large Decision Making Units

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Abstract

In this paper, we develop CCR-(ε-SVR) and CCR-(ν-SVR) models based on modified parameters for efficiency prediction of large DMUs to improve the accuracy and reduce the computation time using three normalization functions. CCR-(*ε*-SVR) and CCR-(*ν*-SVR) are evaluated using large datasets over the three normalization functions. The experimental results of comparisons between CCR-(*ε*-SVR) and CCR-(*ν*-SVR) demonstrate that the proposed models can significantly improve the accuracy and reduce the computation time in predicting the efficiency of large DMUs.

Keywords: Data Envelopment Analysis (DEA), Support Vector Regression (SVR), Large DMUs, Normalization function

1. Introduction

Data Envelopment Analysis (DEA) is an effective method for obtaining the efficiency of Decision Making Units (DMUs) (Charnes et al., 1978; Chen and van Dalen, 2010; Emrouznejad and Shale, 2009). DMUs have been generated based on production function, cost function and measuring efficiency for evaluation and selection (Koopmans, 1951; Debreu, 1951; Farrell, 1957). Conventional DEA methods such as Charnes, Cooper and Rhodes (CCR) and Banker, Charnes and Cooper (BCC) models have been proposed by (Charnes et al., 1978; Banker, et al., 1984). Methods such as Additive DEA model (ADD), Enhanced Russell Measure (ERM) and Slack Based Measure (SBM) have been proposed to classify the DMUs into efficient and inefficient units which are not able to complete ranking for DMUs (Charnes et al., 1985; Färe and Knox, 1978; Pastor et al., 1999). In the most of these DEA models, the best DMUs have an efficiency score of unity, and, the experience is shown that there are usually plural DMUs which have this efficient status.

The dimensions of the original DEA linear programs formulated by Charnes et al. (1987) are essentially the dimensions of the *n* by *m* matrix defined by the data. Since each data point needs to be processed once, the original DEA procedure of Charnes et al. (1987) requires the application of *n* Linear Programs (LPs).

To overcome the computation time in DEA, several researchers tackled the problem in large data sets with using different DMUs (Ali, 1990, 1993, 1994; Ali and Seiford, 1993; Barr et al., 2002; Dulá, 2008).

Although LPs can be solved in polynomial time, the repeated solution of LPs becomes computationally intensive and time consuming, especially when large data sets are involved. In the other hand, for a massive data set with high dimensions of inputs and outputs for DMUs, DEA needs huge computer resources in terms of memory and CPU time (Emrouznejad and Shale, 2009).

The main problems with DEA based methods for large DMUs raking are time consuming and insufficient accuracy. The proposed combination of DEA and SVR (DEA-SVR) method for large DMU's efficiency evaluation was presented by Farahmand et al. (2014). They presented a combination of two conventionally methods, namely CCR and BCC with SVR (CCR/BCC-(ν-SVR)) for measuring of efficiency large DMUs ranking. The data used for training the SVR includes input, output and efficiency of DEA. This method was proposed for efficiency evaluation of large DMUs to solve some drawbacks which include uncontrolled convergence and non-generalization. For SVR, they selected the ν-SVR using RBF kernel and obtained the best value by 2-fold and 10-fold cross-validation. The DEA-SVR method was applied to five data sets each with 5000 units, and compared the results with the combined methods by DEA and NN. Also, Experimental results demonstrated that the proposed method outperforms the earlier developed combined method of back-propagation neural network and DEA.

As can be found from the above literature, the accuracy and computational complexity in measuring the efficiency of large DMUs still have been two main challenges in the field. Besides that, requiring huge computer resources that spend large amount of memory and CPU time for large DMUs is one of the main issues connected to the conventional DEA. In addition, the approaches used in the prior researches have some drawbacks which include uncontrolled convergence, non-generalization and unstable. Thus, in this paper, a comparative study between two new models based on select the best normalization is presented and tested its performance to evaluate efficiencies of large number of DMUs.

The remainder of this paper is organized as follows. Related work of DEA-SVR is presented in Section 2. DEA-SVR method explains in Section 3. Section 4 presents the experimental results and performance evaluation and comparison for the proposed method. Finally, conclusions are presented in Section 5.

2. Related work of DEA-SVR method

2.1 Normalization Functions

The normalization functions and distance measures are also taken into consideration. Four normalization functions extracted from Yoon and Hwang (1995), Milani et al. (2005) and Graf (2001) are used in the experimental study.

1. Normalization (1) (denoted by NF_1)):

$$NF_1 = \frac{x_{ij}}{\max_j \{x_{ij}\}},\ i=1,2,\dots,m,\ j=1,2,\dots,n \tag{1}$$

2. Normalization (2) (denoted by NF_2):

$$NF_2 = \frac{x_{ij} - \min_j \{x_{ij}\}}{\max_j \{x_{ij}\} - \min_j \{x_{ij}\}},\ i=1,2,\dots,m,\ j=1,2,\dots,n \tag{2}$$

3. Normalization (3) (denoted by NF_3):

$$NF_3 = \frac{x_{ij} - \overline{x_{ij}}}{\sigma_j}, i=1,2,\dots,m \ and \ \ j=1,2,\dots,n \tag{3}$$

where σ_j is the standard deviation of alternative ratings with respect to the jth attribute.

2.2 Kernel Functions

The use of a standard kernel functions are as follows (Scholkopf and Smola, 2002; Abe, 2010)):

1. Linear Function (LF):

$$K_1(x_i, x_j) = \langle x_i, x_j \rangle + c \tag{4}$$

where c and $\langle x_i, x_j \rangle$ are products inner between x_i and x_j vectors in R^n and kernel-specific parameters, respectively.

2. Radial Basic Function (RBF):

$$K_2(x_i, x_j) = e^{(-\frac{\|x_i - x_j\|^2}{2\sigma^2})} = e^{-\gamma \|x_i - x_j\|^2} \tag{5}$$

where $\gamma = 1/2\sigma^2$ must be value optimized.

3. Polynomial Function (PF):

$$K_3(x_i, x_j) = (b\langle x_i, x_j \rangle + c)^d \tag{6}$$

where b, c and d are kernel-specific parameters.

4. Sigmoid Function (SF):

$$K_4(x_i, x_j) = \tanh(b\langle x_i, x_j \rangle + c) \tag{7}$$

where b and c are kernel-specific parameters.

5. Cauchy Function (CF):

$$K_5(x_i, x_j) = (1 + \frac{\|x_i - x_j\|^2}{\sigma^2})^{-1} \tag{8}$$

where σ must be value optimized.

6. Logarithm Function (LOGF):

$$K_6(x_i, x_j) = -\log(\|x_i - x_j\|^d + c) \tag{9}$$

Select the parameter optimize such as γ, c, b, d and σ in kernel functions is very important. The parameters are affected in run time and accuracy. In Eq. (5), γ is the important parameter to increase accuracy in SVM algorithm. The best parameters, such as γ in RBF kernel, can be obtained using cross-validation model.

3. DEA-SVR method

There are two SVR models for non-separable data points, namely ε-SVR and ν-SVR. In (Farahmand et al., 2014), ν-SVR model was presented and used in the proposed DEA-SVR method. However, in this paper, two SVR models, namely the ε-SVR and ν-SVR models, are considered for the improved DEA-SVR method. In the following section, the ε-SVR model is presented.

Assume that there are $V_{DMUj}=(X_j, Y_j, \theta_j)$ (for j=1,2,…,n) is vector belongs $R^M \times R$ where, $M=m+s$. A general scheme of the DEA-SVR method with $X_j=(x_{1j}, x_{2j},..., x_{mj})$ and $Y_j=(y_{1j}, y_{2j},..., y_{sj})$ where θ_j, I_i (i=1,2,…,m) and O_r (r=1,2,…,s) are efficiency, input and output vectors of $DMU_j=(X_j,Y_j)$, respectively.

Suppose $(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n) \in R^M \times R$ where R^M is a dot product space and primal objective function for ε-SVR as the following:

$$\min_{w,b} \frac{1}{2} w^t.w + C\sum_{i=1}^{n}(\xi_i+\xi_i^*)$$
$$s.t \begin{cases} y_i-\left(w^t.\phi(x_i)+b\right) \le \varepsilon+\xi_i \\ \left(w^t.\phi(x_i)+b\right)-y_i \le \varepsilon+\xi_i^* \\ \xi_i, \xi_i^* \ge 0, \quad i=1,2,\ldots,n \end{cases} \tag{10}$$

where $C > 0$, $\varphi(x_i)$ maps x_i into a higher dimensional space, w is the margin.

Dual optimization problem of ε-SVR model is given by:

$$\begin{cases} \max\limits_{\alpha} & -\frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k}\phi(x_i)\phi(x_j)(\alpha_i-\alpha_i^*)(\alpha_j-\alpha_j^*)-\varepsilon\sum_{i=1}^{k}(\alpha_i+\alpha_i^*)-\sum_{i=1}^{k}y_i(\alpha_i-\alpha_i^*) \\ & subject\,to: \sum_{i=1}^{k}(\alpha_i-\alpha_i^*)=0 \quad and \quad \alpha_i, \alpha_i^* \in [0,C] \end{cases} \tag{11}$$

The prediction function is given as,

$$f(x)=\sum_{i=1}^{n}(\alpha_i-\alpha_i^*)K(x,x_i)+b \tag{12}$$

where,

$$K(x_i,x_j)=\phi(x_i)\phi(x_j)=Q_{ij} \tag{13}$$

K is a kernel function and Q is called kernel matrix and it is used in the SVR algorithm. In this chapter, the following RBF kernel function is used.

3.1 Proposed DEA-SVR method

In this section, the proposed combination of DEA and SVR, DEA-SVR, method for DMU's efficiency evaluation is presented. The ν-SVR model is presented and used in the new proposed DEA-SVR method by Farahmand et al. (2014). In this proposed method, the authors are testing the two common methods of DEA, namely CCR and BCC (Cooper et al., 2007), and combined them individually with SVR. The DEA-SVR method studied consist of CCR-SVR and BCC-SVR methods. In this paper, two SVR models, namely the ε-SVR and ν-SVR models are considered for the improved CCR-SVR method called CCR-(ε-SVR) and CCR-(ν-SVR), respectively. The process of DEA-SVR method is presented in Algorithm 1 as following (Farahmand et al., 2014):

Algorithm 1: Process of DEA-SVR method

Step 1: Get the input data=(X_j,Y_j)=[DMU Inputs|DMU Outputs]. The method requires a set of DMUs, each with set of DMU's input and output.
Step 2: Calculate efficiency of units using DEA with CCR model (Obtained θ_{CCR}).
Step 3: Calculate NIO=Normalized [DMU Inputs|DMU Outputs].
here, normalization functions Eq.(1,2 and 3) are used.
Step 4: Calculate X_{CCR}=[NIO|θ_{CCR}].
Step 5: Calculate $XSVR_{CCR}$=[feature column number: X_{CCR}]
where the number of features = the number of DMU inputs + the number of DMU outputs.
Step 6: Calculate (Feature set) FS_{CCR}= [θ_{CCR} |$XSVR_{CCR}$].
Step 7: Select the best parameters C, γ ,ν , ε and the best kernel function for ν-SVR or ε-SVR model based on t run programming (t trials) by do k-Fold cross-validation (obtain MSE average for each run and select the best solution).
Step 8: If the results are satisfactory then stop, else go to Step 7.
Step 9: Stop.

3.2. Proposed CCR-(ε-SVR) and CCR-(ν-SVR) models

There are two SVR models for non-separable data points, namely ε-SVR and ν-SVR. In algorithm 1, DEA and SVR are replaced with CCR and ε-SVR or ν-SVR to develop two hybrid methods called CCR-(ε-SVR) and CCR-(ν-SVR), respectively. The CCR-(ν-SVR) model was developed by Farahmand et al. (2014) based on RBF kernel and normalization function as presented in Eq. (5) and Eq. (2), respectively. In this paper, we develop CCR-(ν-SVR) and CCR-(ε-SVR) model based on 4 normalization functions and other kernel functions in SVR. The results showed that RBF function is better than other functions for predicting efficiency of large DMUs. In this paper, NF_1, NF_2 and NF_3 were selected to be used in algorithm 1. Furthermore, selecting parameters of ε-SVR or ν-SVR are very important in improvement of CCR-(ε-SVR) or CCR-(ν-SVR) models. Therefore, selecting the suitable kernel function, normalization function, SVR models and their parameters can play important role in achieving noticeable results.

4. Experimental results and performance comparisons

This section explains the experiments carried out to test the performance of the proposed DEA-SVR algorithm for large DMU's data sets. The performance is in terms of accuracy which is measured by several evaluation methods. Performance comparisons between normalization functions, DEA-SVR models, predicting efficiency of new DMUs and stability analysis of DEA-SVR with DEA-NN are also presented.

4.1 Evaluation methods

In this section, the experiments perform on large data sets to test the performance of the proposed method. The performance is defined in terms of accuracy which is measured by reduction of efficient units to obtain the DMU's ranking. In our experiments, performance comparisons between proposed method and CCR model and integrated DEA with prediction methods are presented. For evaluating the proposed method, one measures of accuracy are used to determine the algorithm capability for DMUs ranking.

***K*-Fold cross-validation:** *K*-Fold cross-validation estimates of performance cross-validation is a computer intensive technique, using all available examples as training and test examples. It makes pattern for the use of training and test sets by repeatedly training the algorithm *K* times with a fraction 1/*K* of training examples left out for testing purposes. This kind of hold-out estimate of performance lacks computational efficiency due to the repeated training, but the latter are meant to lower the variance of the estimate. Cross Validation metrics are used based on as following (Bengio and Grandvalet, 2004):

- Mean Squared Error (MSE):

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(f(x_i) - y_i)^2 \qquad (14)$$

where $f(x_i)$ must be approximated by y_i.

Root Mean Squared Error RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f(x_i) - y_i)^2} \qquad (15)$$

where $f(x_i)$ must be approximated by y_i.

4.2 Data set description

The experimental data set for this experiment was taken from (Farahmand et al., 2014) which consist of 5 data sets. Each data set contains 5000 units and each unit has 6 attributes of 3 inputs and 3 outputs. The program code for DEA-SVR (i.e. CCR-(ε-SVR) and CCR-(ν-SVR)) and DEA-NN (i.e. CCR-NN) are improved by LIBSVM (Chang and Lin, 2011) implemented in MATLAB software.

Furthermore, selecting parameters of ε-SVR or ν-SVR are very important in improvement of CCR-(ε-SVR) or CCR-(ν-SVR) models. As a result, selecting the suitable kernel function, normalization function, SVR models and their parameters can play important role in achieving noticeable results.

4.3 Comparison of results between CCR-(ε-SVR) and CCR-(ν-SVR) models

In this section, the data sets have been used for evaluating the proposed method using large DMUs (DATA 1-5). In the following, the experiments for efficiency and effectiveness evaluations of proposed methods are provided. Firstly, accuracy of the proposed method is evaluated using large data sets for predicting the efficiency of large DMUs.

Table 1 shows the results of comparison between CCR-(ε-SVR) and CCR-(ν-SVR) models based on accuracy (MSE), standard deviation (SD) and time average (TA (seconds)) for predicting the efficiency of large DMUs using DATA 1-5. In this table, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This table showed NF_3 is better than other normalization functions in terms of MSE, SD and TA.

Table 2 shows the results of comparison between CCR-(ε-SVR) and CCR-(ν-SVR) models based on MSE, SD and time average (TA (seconds)) for predicting the efficiency of large DMUs using DATA 1-5. In this table, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This table showed NF_3 is better than other normalization functions in terms of MSE.

Fig. 1 shows the results of comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ε-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 2-Fold cross-validation for predicting the efficiency of large DMUs using DATA 1-5. In this figure, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This figure showed NF_3 is better than other normalization functions in terms of TA and MSE.

Table 1
A comparison between CCR-(ε-SVR) and CCR-(ν-SVR) models with 3 functions (NF_1, NF_2 and NF_3) based on accuracy, standard deviation (SD) and computational time using prediction of large DMUs for DATA 1-5.

10 Trials, 2-Fold	CCR-(ε-SVR)			CCR-(ν-SVR)		
Normalization Functions	NF_1 C^*=2000 γ^* =3	NF_2 C^*=3000 γ^* =3	NF_3 C^*=1000 γ^*=0.3	NF_1 C^*=2000 γ^* =3	NF_2 C^*=3000 γ^* =3	NF_3 C^*=1000 γ^*=0.3
DATA 1						
MSE (%)	15.0884	14.8467	14.2786	14.0061	14.5463	13.9577
SD	0.9843	0.8832	0.8523	1.2307	0.6244	1.0374
TA (seconds)	37.5366	64.3017	30.0233	68.4814	74.4167	49.4870
DATA 2						
MSE (%)	14.5529	15.0613	13.9652	14.1861	14.5483	13.7174
SD	1.0067	1.1318	1.1593	1.3560	0.9645	0.9755
TA (seconds)	46.8041	77.4965	32.5664	75.6564	87.66	62.3933
DATA 3						
MSE (%)	16.0605	16.4356	14.7559	15.268	16.2436	14.3155
SD	1.0768	1.3887	1.1703	1.2722	1.0358	0.8098
TA (seconds)	40.4969	72.0403	30.3113	73.6281	82.2234	62.3122
DATA 4						
MSE (%)	20.67	20.7994	19.4043	20.284	20.463	20.4606
SD	1.8169	1.2722	1.6367	1.5092	1.6196	1.8233
TA (seconds)	38.8590	67.0582	29.8013	73.5659	82.2743	50.8884
DATA 5						
MSE (%)	16.3075	16.2085	15.3055	15.1061	15.8147	15.1959
SD	0.7844	1.3809	1.33	0.9576	1.4301	1.2848
TA (seconds)	42.1743	74.2466	32.9482	74.2050	83.3519	58.316

Table 2
A comparison between CCR-(ε-SVR) and CCR-(ν-SVR) models with 3 functions (NF_1, NF_2 and NF_3) based on accuracy, standard deviation (SD) and computational time using prediction of large DMUs for DATA 1-5.

10 Trials, 5-Fold	CCR-(ε-SVR)			CCR-(ν-SVR)		
Normalization Functions	NF_1 C^*=2000 γ^* =3	NF_2 C^*=3000 γ^* =3	NF_3 C^*=1000 γ^*=0.3	NF_1 C^*=2000 γ^* =3	NF_2 C^*=3000 γ^* =3	NF_3 C^*=1000 γ^*=0.3
DATA 1						
MSE (%)	12.3265	11.7959	10.6495	12.0632	11.6315	10.835
SD	1.2533	1.1184	1.4	1.36	1.3275	1.3235
TA (seconds)	83.9220	150.5971	87.2981	91.2694	169.6276	109.6831
DATA 2						
MSE (%)	12.3182	12	10.6774	12.0447	11.8793	10.7054
SD	1.3925	1.2803	1.3768	1.433	1.3211	1.5379
TA (seconds)	73.0046	132.3523	101.1241	104.1306	199.0474	145.5955
DATA 3						
MSE (%)	13.8045	13.3739	11.4286	13.5515	13.309	11.4787
SD	1.5150	1.7099	1.4434	1.4708	1.4718	1.9036
TA (seconds)	66.2379	124.1739	100.5981	100.7983	188.2306	137.4871
DATA 4						
MSE (%)	17.7145	17.3034	16.0590	17.3908	17.264	16.185
SD	3.4534	2.9239	1.4682	2.1944	2.0506	2.0299
TA (seconds)	70.8754	126.9968	88.7617	101.5679	184.7574	117.9215
DATA 5						
MSE (%)	13.5863	13.3267	11.7044	13.4138	13.096	11.6743
SD	2.2909	1.6528	1.2909	1.3837	1.8950	1.3799
TA (seconds)	69.6023	126.8336	97.8541	102.7346	188.7672	134.1743

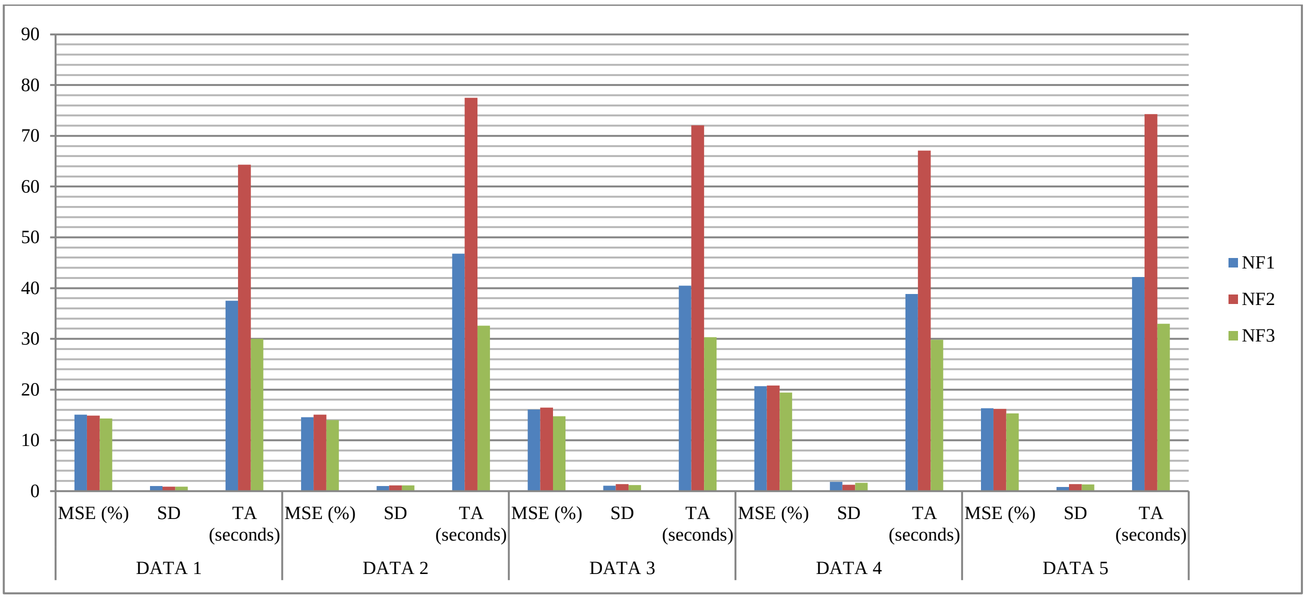


Fig. 1 A comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ε-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 2-Fold cross-validation.

Fig. 2 shows the results of comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ν-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 2-Fold cross-validation for predicting the efficiency of large DMUs using DATA 1-5. In this figure, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This figure showed NF_3 is better than other normalization functions in terms of TA and MSE.

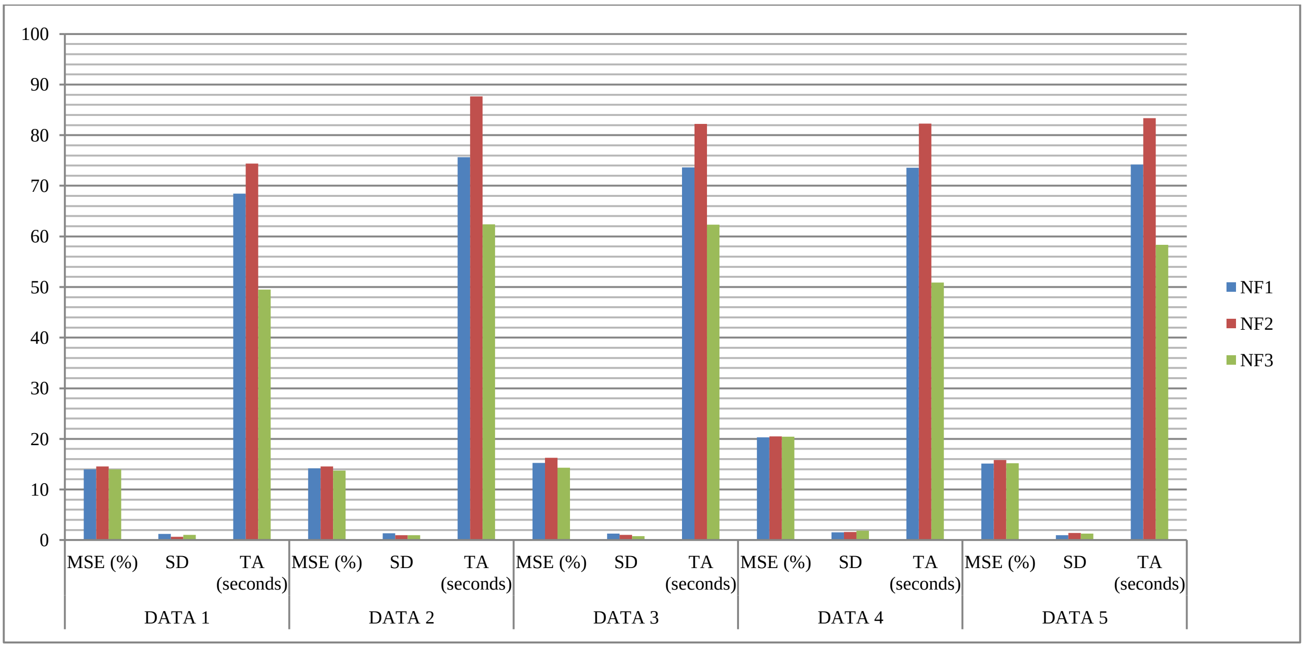


Fig. 2 A comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ν-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 2-Fold cross-validation.

Fig. 3 shows the results of comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ε-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 5-Fold cross-validation for predicting the efficiency of large DMUs using DATA 1-5. In this figure, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This figure showed NF_1 is better than other normalization functions in terms of TA and MSE.

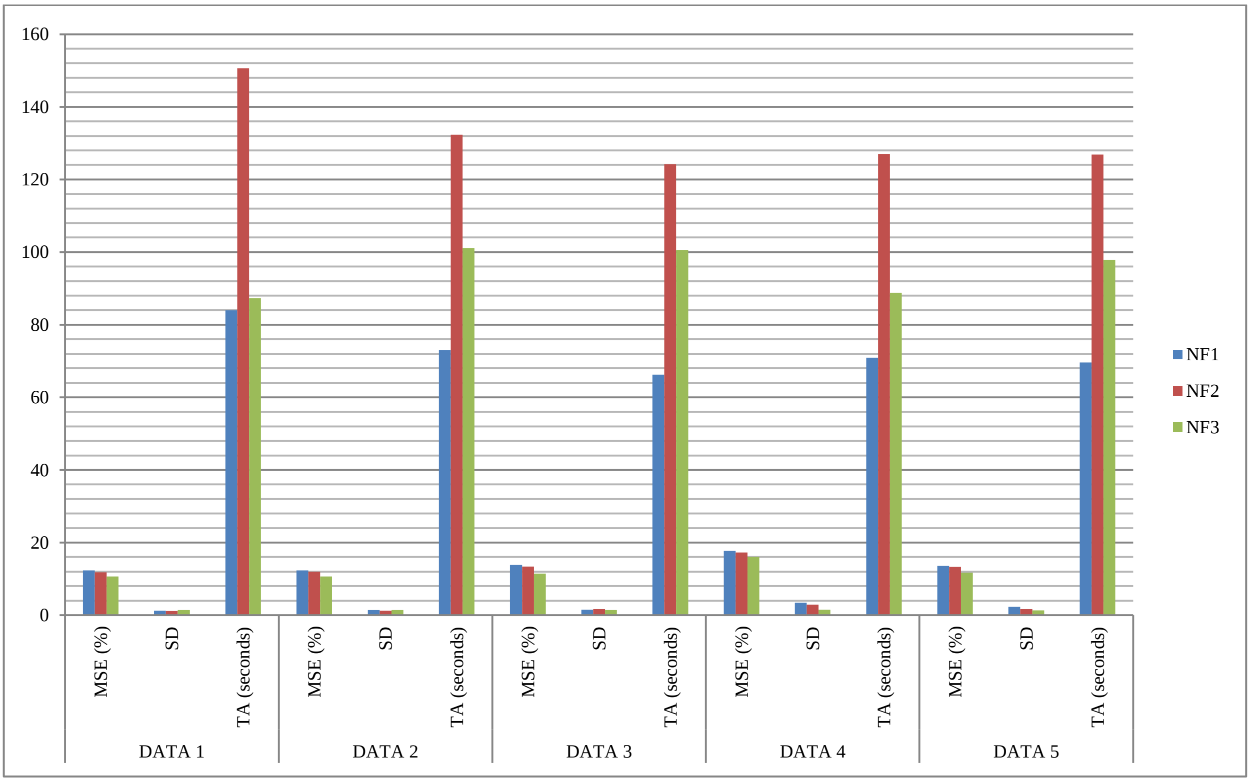


Fig. 3 A comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ε-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 5-Fold cross-validation.

Fig. 4 shows the results of comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ν-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 5-Fold cross-validation for predicting the efficiency of large DMUs using DATA 1-5. In this figure, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This figure showed NF_1 is better than other normalization functions in terms of TA and MSE.

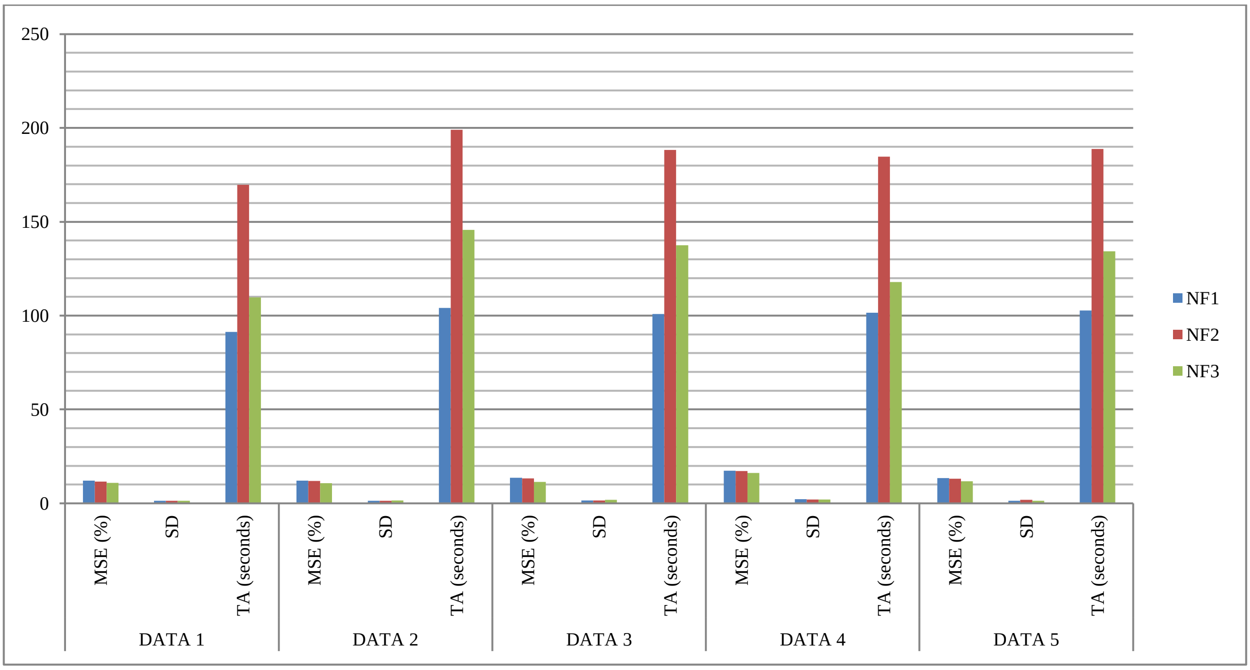
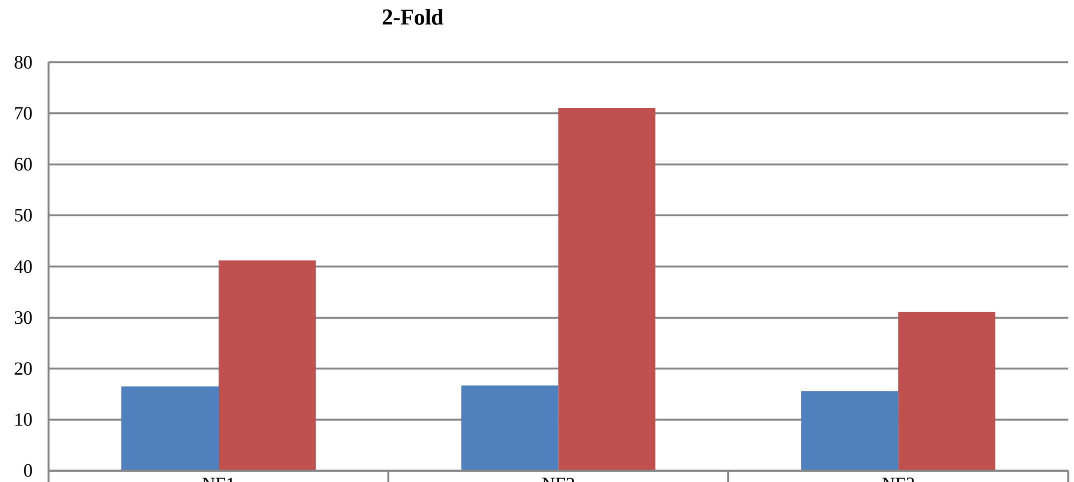


Fig. 4 A comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ν-SVR) model based on MSE, Standard Deviation (SD) and TA (seconds) for DATA 1-5 with 5-Fold cross-validation.

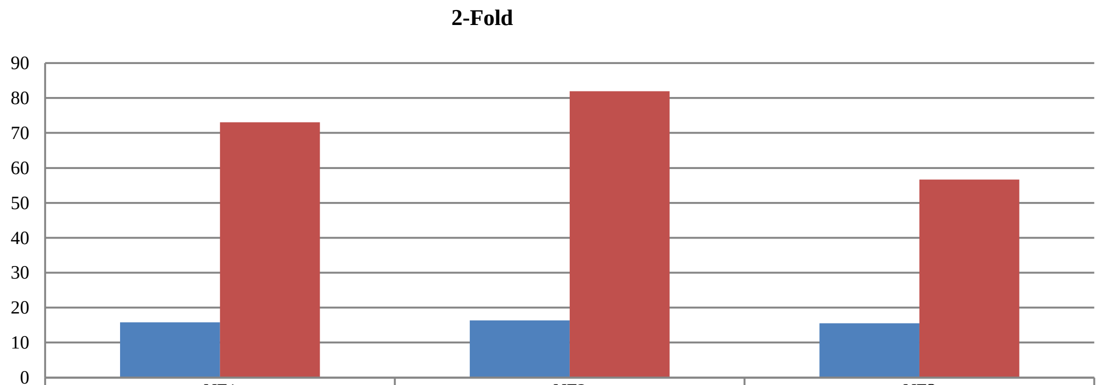
Fig. 5 shows the results of comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ε-SVR) model based on average of MSE, and TA (seconds) for DATA 1-5 with 2-Fold cross-validation for predicting the efficiency of DMUs using DATA 1-5. In this figure, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This figure showed NF_3 is better than other normalization functions in terms of TA.



	NF1	NF2	NF3
MSE AVERAGE	16.53586	16.6703	15.5419
TA AVERAGE	41.17418	71.02866	31.1301

Fig. 5 A comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ε-SVR) model with 2-Fold cross-validation based on average of MSE and TA for DATA 1-5.

Fig. 6 shows the results of comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ν-SVR) model based on average of MSE, and TA (seconds) for DATA 1-5 with 2-Fold cross-validation for predicting the efficiency of DMUs using DATA 1-5. In this figure, results are obtained based on three types of best normalization functions namely NF_1, NF_2 and NF_3 with RBF kernel. This figure showed NF_3 is better than other normalization functions in terms of TA. Fig. 7 shows the results of comparison between CCR-(ε-SVR) and CCR-(ν-SVR) models using based on average of MSE and TA (seconds) for DATA 1-5 with 2-Fold cross-validation for predicting the efficiency of DMUs using DATA 1-5. In this figure, results are obtained based on NF_3 and RBF kernel. This figure showed CCR-(ε-SVR) model is better than CCR-(ν-SVR) models in terms of average of TA for DATA 1-5.



	NF1	NF2	NF3
MSE AVERAGE	15.77006	16.32318	15.52942
TA AVERAGE	73.10736	81.98526	56.67938

Fig. 6 A comparison between 3 functions NF_1, NF_2 and NF_3 using CCR-(ν-SVR) model with 2-Fold cross-validation based on average of MSE and TA for DATA 1-5.

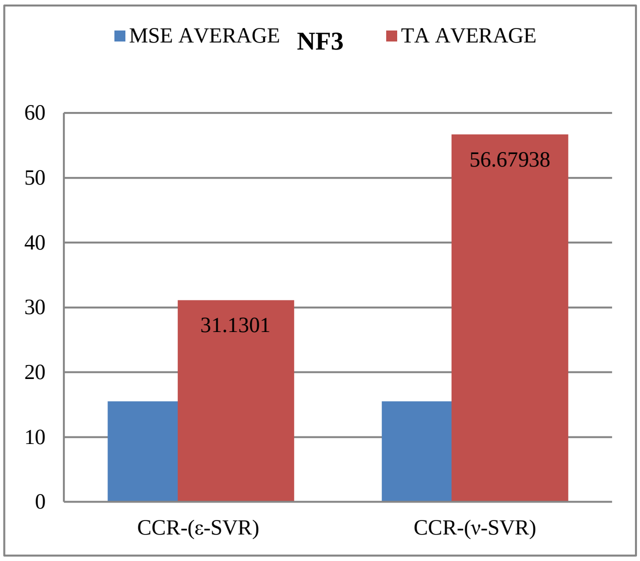


Fig. 7 A comparison between CCR-(ε-SVR) and CCR-(ν-SVR) models using NF_3 using with 2-Fold cross-validation based on average of MSE and TA for DATA 1-5.

5. Conclusions

This paper developed Data Envelopment Analysis (DEA) and Support Vector Regression (SVR) for improving CCR-(ν-SVR) and CCR-(ε-SVR) models using 4 normalization functions. The results showed that Radial Basic Function (RBF) kernel is better than other functions for predicting efficiency of large DMUs. The study showed that selecting parameters of ε-SVR or ν-SVR is very important in improvement of DEA-SVR method. In addition, the results showed that selecting the suitable kernel function, normalization function, SVR models and their parameters plays an important role in achieving high accuracy improvement and low computation time. Finally, CCR-(ν-SVR) model solved problems such as instability, computational time and insufficient accuracy for predicting efficiency of new small and large DMUs with new proposed normalization functions.

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