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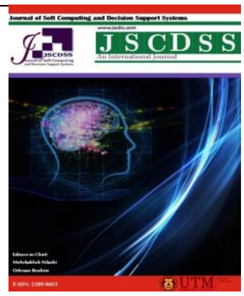

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Design a Tracking Control Law for Nonlinear Continuous Time Fuzzy Polynomial Systems

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Abstract

In this paper, the method of designing a tracking control law for fuzzy polynomial systems is investigated. In the proposed method, which is a generalization of the existing methods, the nonlinearities of the system are also considered. Therefore, a wide range of systems can be controlled using this approach. The output feedback control rule is considered based on the structure of the observer and the controller. The closed-loop system stability and performance conditions will be extracted in the format of the sum of squares by guaranteeing H_∞ tracking index. The proposed method is a generalization of the tracking control law design for Sugeno fuzzy systems. The numerical simulation results show the performance of the proposed method.

Keywords: Polynomial fuzzy systems, Tracking control rule, Sum of squares observer controller, Infinite norm, Takagi-Sugeno, Nonlinear continues time.

1. Introduction

The Takagi-Sugeno (T-S) fuzzy model has received much attention in recent years after applying linear matrix inequality, and so far, much research has been done, especially on the application of linear matrix inequality in various fields like optimal control, robust control and non-linear control (Wang et al., 1995). In general, the control method based on the fuzzy model offers a straightforward and effective method as a complement to other non-linear control methods. Although linear matrix inequality control is an efficient and effective method for the T-S fuzzy model, many design problems cannot be articulated by linear matrix inequality.

In recent years, another set of fuzzy systems has been developed that are more general in modelling non-linear systems than T-S fuzzy systems. This class of systems are known as Fuzzy Polynomial Systems (FPS). The main difference between these systems and the T-S fuzzy systems is that the linear subsystems in the T-S system rules section have become polynomial subsystems dependent on the system states (Tanaka et al., 2006). Various activities have been performed to analyse the fuzzy polynomial systems. Among these are (Yu et al., 2018; Izadi and Ghasemi, 2019; Izadi et al., 2014; Yan et al., 2019; Shahri et al., 2019). In addition, this control method can also be used for non-linear systems such as heterogeneous traffic networks in order to maximize the outflow of the highway (Tanaka et al., 2012).

Stabilization and tracking are two primary aims for control problems, which the later one is more difficult to pursue than stabilization. There are various methods for designing the output tracking control rule in the literature, such as the observer-controller design. Some work has been done on the observer design to stabilize polynomial fuzzy systems (e.g., Chang and Wu, 2012). In this research, polynomial fuzzy observer for three classes of polynomial fuzzy systems is investigated. As mentioned, the results of the study in Tanaka et al. (2006) are solely dedicated to system stabilization and do not apply to the tracking problem.

Various studies have been carried out on the design of the observer for T-S fuzzy systems. Investigation of these studies will allow the development of the methods of fuzzy polynomial systems. In Shanjian et al. (2015), a fuzzy controller and observer are designed to reduce the tracking error. In this paper, H∞ scheme is used to design the observer gain. In Tseng et al. (2001), the problem model reference tracking control for discrete-time affine fuzzy systems with external disturbances is introduced, which uses H∞ scheme to reduce the disturbance effect (e.g., Chang and Wu, 2012). In this paper, based on the fuzzy

Lyapunov functions, the necessary conditions are derived to obtain the control law for the tracking problems. In Park et al. (2002), a method for the design of fuzzy control for non-linear systems with a guaranteed model reference tracking performance investigated. It is designed to reduce the tracking error of a fuzzy observer with a fuzzy controller for the primary system. The conditions in this design are presented based on linear matrix inequality. The design of the adaptive fuzzy-model-based controller for chaotic dynamics in Lorenz systems with uncertainty and time delay is discussed in Izadi et al. (2017). The proposed method in Izadi et al. (2017) is like Park et al. (2002), except that it has been studied for discrete-time systems with time delay. In the design process of a supervisory software algorithm that can detect and classify different types of faults and determine the fault source (Izadi et al., 2016; Tanaka et al., 2007), the polynomial fuzzy systems can be used.

In this paper, the method outlined in Park et al. (2002) for the design of polynomial fuzzy control law is generalized. The conditions proposed to guarantee the stability and performance of the closed loop system are similar in the first theory, but in this work, we are expanding the theory. In addition, according to the extracted conditions for controller and observer derivation are generally non-convex, they have been modified by criticizing the method presented in Park et al. (2002) and expressing its shortcomings.

In the remainder of this paper, the problem definition and basic mathematics are presented in Section 2. Section 3 proposes the requirements for designing the control law and polynomial fuzzy observer design. In Section 4, the performance of the method is shown using the numerical simulation for two examples system. The fifth Section is devoted to summarizing and concluding the paper.

2. Problem Statement

2.1 *Basic Mathematics*

The monotonic function $x(t)$is a function which has one term of the variable $x(t)$. A monotonic function can be a number, a variable or product one or more primes in a variable in the form of $\beta\big(x(t)\big)^{\alpha}$, where α and β are numbers. A polynomial function $f\big(x(t)\big)$ is a function that results from the sum of several monotonic functions of $x(t)$. The polynomial function $f\big(x(t)\big)$ is the summation of squares if the polynomials $f_1\big(x(t)\big)$, ... , $f_k\big(x(t)\big)$ exist such $f\big(x(t)\big) = \sum_{i=1}^{k} f_i^2\big(x(t)\big)$; the sum of the squares for the $f\big(x(t)\big)$ means that for $x(t) \in \mathbb{R}^n$ we have $f\big(x(t)\big) \geq 0$. Suppose a nonlinear system as:

$$\dot{x}\ (t)=(x(t),u(t))+\omega(t) \tag{1}$$

where f is a nonlinear function by $f(0,0) = 0$. $x(t) = [x_1(1)\ x_2(t)\ \dots\ x_n(t)]^t$ is the state vector, $u(t) = [u_1(1)\ u_2(t)\ \dots\ u_n(t)]^T$ is input vector and $\omega(t) = [\omega_1(1)\ \omega_2(t)\ \dots\ \omega_n(t)]^T$ is external disturbance. The Eq. (1) can be written in the form of a fuzzy polynomial by using the sector nonlinearity method in Huang et al. (2007).

$$\begin{aligned}&if\ z_1(t)\ is\ M_{i1}\ and\ \cdots\ and\ z_p(t)\ is\ M_{ip}\\ &Then\ \dot{x}(t) = A_i\big(x(t)\big)x(t) + B_i\big(x(t)\big)u(t) + \omega(t)\end{aligned} \tag{2}$$

where $i = 1,2,\dots,r$. In Eq. (2) r represents the number of rules. For the i^{th} model rule and j^{th} variable, the membership function is defined by M_{ij}. Each $z_j(t)$ is a time variant parameter which can be a state, a measurable variable and/or time. $B_i(x(t)) \in \mathbb{R}^{n\times m}$ and $A_i(x(t)) \in \mathbb{R}^{n\times n}$ are polynomial matrices of $x(t)$. Also, $A_i\big(x(t)\big)x(t) + B_i\big(x(t)\big)u(t) + \omega(t)$ is a polynomial vector. Therefore, the fuzzy model in Eq. (2) in its section contains a polynomial function. The mathematical form of Eq. (2) can be described in the following state model:

$$\begin{aligned}&\dot{x}(t) = \sum_{i=1}^{r} h_i\big(z(t)\big)A_i\big(x(t)\big)x(t) + B_i\big(x(t)\big)u(t) + \omega(t)\\ &y = \textstyle\sum_{i-1}^{r} h_i\big(z(t)\big)C_i x(t) + \vartheta(t)\end{aligned} \tag{3}$$

2.2 *Mathematical construction of the problem*

Consider the following reference model:

$$\dot{x}(t) = A_r x(t) + B_r u(t) \tag{4}$$

where A_r and B_r are reference model matrices of the system and $r(t)$ is reference input. It must be considered that in Park et al. (2002) the matrix B_r is in the unity form, while here we supposed a more general form for that. The index of the H∞ norm related to the tracking error $x(t) - x_r(t)$ will be (Park et al., 2002):

$$\begin{aligned}&\int_0^{t_f} [x(t) - x_r(t)]^T Q[x(t) - x_r(t)]\}\,dt\\ &\qquad \leq \rho^2 \textstyle\int_0^{t_f} \tilde{\omega}(t)^T \tilde{\omega}(t)dt\end{aligned} \tag{5}$$

where $\tilde{\omega}(t) = [\vartheta(t)\ \ \omega(t)\ r(t)]^T$ and $r(t)$ is reference input, $\omega(t)$ is the external disturbance, $v(t)$ is measurement noise. In Eq. (5) t_f is the control termination time, Q is a positive definite weight matrix and ρ is attenuation value. From a practical sense, the Eq. (5) shows that the effect of $\tilde{\omega}(t)$ on the tracking error $x(t) - x_r(t)$ in the aspect of the energy level is less than predefined value for ρ. In other words, gain L_2 in $\tilde{\omega}(t)$ over $x(t) - x_r(t)$ must be equal or less than attenuation gain ρ. In order to estimate the states of the system in the Eq. (3) by using polynomial fuzzy observer we have:

$$\begin{aligned}&\hat{x}(t) = \sum_{i=1}^{r} h_l z\{A_l(\hat{x})x + B_l(\hat{x})u + L_l(\hat{x})(y - \hat{y})\\ &\hat{y} = \sum_{i=1}^{r} h_i(z)C_i\hat{x}\end{aligned} \tag{6}$$

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The estimation error is the difference between the states of the original system and those of the observer. Derivation of this estimation error will be:

$$e = x - \hat{x}$$
$$\dot{e} = \dot{x} - \dot{\hat{x}}$$

$$\sum_{i=1}^{r} h_i z \sum_{j=1}^{r} h_j z = [A_i(x) + B_i(x)u + \omega] - [A_i(\hat{x})x + B_i(\hat{x})u + L_i(\hat{x})C_j(x-\hat{x}) + L_i(\hat{x})\vartheta]$$

$$\sum_{i=1}^{r} h_i z \sum_{j=1}^{r} h_j z\,[(A_i(\hat{x}) - L_i\hat{x}C_j)e + (A_j(\hat{x}) - A_i(\hat{x}))x - L_i(\hat{x}\vartheta)] \quad (7)$$

3. Design of control tracking law

For stabilizing and tracking the control system in Eq. (3), the control law is as follow:

$$u = \sum_{j=1}^{r} h_j(z)[K_j(\hat{x})(\hat{x} - x_r)] \quad (8)$$

As it was discussed before, the main goal of this research is to reduce the tracking error, so in the control law in Eq. (8), we have the term, which shows the difference between the observer state, and the reference state $x(t) - x_r(t)$ in which it helps to reduce the input tracking error by stabilizing the system with control law. It is important to note that since the original states of the system x may not be available, we need to use observer state in the control law. The control law in Eq. (8) is as like as the control law presented in Tseng et al. (2001) except that in Eq. (8) the coefficient K_j is a polynomial matrix of observable states, which it means it includes the state that is more general.

According to Tseng et al. (2001), a system has been added in which the estimation error in Eq. (7) and the reference model in Eq. (1) are stabilized by the control law Eq. (8) to minimize the estimation error of the system and to reduce the error of the tracking error. Therefore, by replacing the control law with u and using the estimation error in Eq. (7) and the reference model in Eq. (4) we have the following equation:

$$\dot{\tilde{x}} = \sum_{i=1}^{r} h_i(z) \sum_{j=1}^{r} [\tilde{A}_{ij}\tilde{x} + \tilde{E}_i\tilde{w}] \quad (9)$$

$$\tilde{A}_{ij} = \begin{bmatrix} A_i(\tilde{x}) - L_i(\tilde{x})C_j & A_i(x) - A_i(\tilde{x}) & 0 \\ -B_i(x)K_j & A_i(x) + B_i(x)K_j & -B_i(x)K_j \\ 0 & 0 & A_r \end{bmatrix}$$

$$\tilde{x} = \begin{bmatrix} e \\ x \\ x_r \end{bmatrix}, \quad \tilde{\omega} = \begin{bmatrix} v \\ \omega \\ r \end{bmatrix}, \quad \tilde{E} = \begin{bmatrix} -L_i(\hat{x}) & I & I \\ 0 & I & 0 \\ 0 & 0 & B_r \end{bmatrix}$$

Since this paper considers the matrix A_i a polynomial and it is related to the states (x), the matrix obtained in this paper is different from the one which is obtained in Tseng et al. (2001). H_∞ tracking norm index is:

$$\int_0^{t_f} \{[x - x_r]^{\mathrm{T}} \mathrm{Q}[x - x_r]\mathrm{dt}\} = \int_0^{t_f} \{\tilde{x}^T \tilde{Q}\tilde{x}dt\} \le \tilde{x}^T(0)\tilde{P}\tilde{x}(0) + \rho^2 \int_0^{t_f} \tilde{\omega}^T\tilde{\omega}dt \quad (10)$$

in which P is the positive symmetric weight matrix and:

$$\tilde{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & Q & -Q \\ 0 & -Q & Q \end{bmatrix}$$

The main purpose of this paper is to determine the polynomial fuzzy controller (see Eq. (8)) for the augmented system (see Eq. (9)) by guaranteeing H_∞ tracking norm index for all $w(t)$. Then the attenuation rate of ρ can be minimized until H_∞ tracking norm index in Eq. (10) is reduced as much as possible.

Theorem 1: in the nonlinear system if we have $p = p > 0$, the inequality matrix is:

$$\tilde{A}_{ij}(x,\tilde{x})\tilde{P}(x) + P(x)\tilde{A}_{ij}(x,\tilde{x}) + \frac{1}{\rho^2}\tilde{P}(x)\tilde{E}_i(\hat{x})\tilde{E}_i(\hat{x})^T\tilde{P}(x) + \tilde{Q} \le 0 \quad (11)$$

If we have $h_i(z), h_j(z) \neq 0$ for $i,j = 1,2,\dots,L$ then the H_∞ tracking norm index in Eq. (10) is guaranteed for ρ. In Eq. (11), E_i and A_{ij} matrices can be in polynomial form. Because the matrices used here are polynomials, therefore they must be written in terms of the sum of squares. For the proof, refer to Tseng et al. (2001) except that the matrices used in this article are polynomials. In order to design easier, we suppose that:

$$\tilde{P}_x = \begin{bmatrix} \tilde{P}_{11}(x) & 0 & 0 \\ 0 & \tilde{P}_{22}(x) & 0 \\ 0 & 0 & \tilde{P}_{33}(x) \end{bmatrix} \quad (12)$$

By substituting Eq. (12) into Eq. (11) it can be indicated that:

$$\begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & S_{23} \\ 0 & S_{32} & S_{33} \end{bmatrix} \quad (13)$$

in which:

$$S_{11} = \left(A_i(\hat{x}) - L_i(\hat{x})C_j\right)^T \tilde{P}_{11}^T(x) + \tilde{P}_{11}(x)(A_i(\hat{x} - L_i(\hat{x})) + \frac{1}{\rho^2}\tilde{P}_{11}(x)(L_i(\hat{x})L_i(\hat{x})^T + I)\tilde{P}_{11}(x)$$

$$S_{12} = S_{21}^T = -\left(B_iK_i(\hat{x})\right)^T \tilde{P}_{22}^T(x) + \tilde{P}_{11}(x)(A_i(\hat{x}) - A_i(\hat{x})) + \frac{1}{\rho^2}\tilde{P}_{11}(x)\tilde{P}_{22}(x)$$

$$S_{23} = S_{32}^T = -\tilde{P}_{22}(x)B_i(x)K_j(\hat{x}) - Q$$

$$S_{33} = A_r^T \tilde{P}_{33}(x) + \tilde{P}_{33}(x) A_r + \frac{1}{\rho^2} \tilde{P}_{33}(x) B_r B_r^T \tilde{P}_{33}$$

By considering $Z_i = P_{11} L_i(x)$ we have:

$$\begin{bmatrix} M_{11}^* & \tilde{P}_{11} & Z_i & {M_{14}^*}^T & 0 & 0 \\ \tilde{P}_{11} & -\rho^2 I & 0 & 0 & 0 & 0 \\ Z_i^T & 0 & -\rho^2 I & 0 & 0 & 0 \\ M_{41}^* & 0 & 0 & M_{44}^* & M_{45}^* & 0 \\ 0 & 0 & 0 & M_{54}^* & M_{55}^* & \tilde{P}_{33} \\ 0 & 0 & 0 & 0 & \tilde{P}_{33} & -\rho^2 I \end{bmatrix} \quad (14)$$

so that:

$$M_{11}^* = A_i(\hat{x})^T \tilde{P}_{11}(x) A_i(\hat{x}) - Z_i(x,\hat{x}) C_j - Z_i(x,\hat{x}) C_j^T$$
$$M_{41}^* = M_{14}^* = \left(A_i - A_i(\hat{x})\right)^T \tilde{P}_{11}(x) - \tilde{P}_{22}(x) B_i(x) K_j(\hat{x}) + \frac{1}{\rho^2} \tilde{P}_{11}(x) \tilde{P}_{22}(x)$$
$$M_{45}^* = M_{54}^* = -\tilde{P}_{22}(x) B_i(x) K_j(\hat{x}) - Q$$
$$M_{55}^* = A_r^T \tilde{P}_{33}(x) + P_{33}(x) A_r + Q$$

It should be noted that the Eq. (14) is not a sum of squares because it is not convex. In the Eq. (14), we have 5 unknown parameters L_i, K_j, P_{11}, P_{22} and P_{33} such that according to the Eq. (14) they are not computable by their self. In Tseng et al. (2001), in order to solve this problem and solve the inequality relation of the linear matrix, the necessary condition to establish Eq. (14) is $M_{44} < 0$, so that we have:

$$\left(A_i(x) + B_i(x) K_j\right)^T \tilde{P}_{22} + \tilde{P}_{22}\left(A_i(x) + B_i(x) K_j\right) + \frac{1}{\rho^2} \tilde{P}_{22} \tilde{P}_{22} + Q < 0 \quad (15)$$

Then by using Eq. (15), K_j and P_{22} parameters can be calculated and by replacing these parameters in Eq. (14), the obtained equation is deduced to a standard linear inequality matrix and also P_{11}, P_{33}, L_i parameters can be obtained so as a result, the observer and controller coefficients can be calculated.

The method presented in Tseng et al., (2001) has one problem which is that the K_j controller coefficients can be obtained by solving the Eq. (15). It is obvious from Eq. (15) that the coefficients obtained from the controller are independent of the specifications defined for the reference model. To solve this problem, in this paper, the Eq. (14) is a sum of squared unless the matrix and the decision variable are considered.

4. Simulation

4.1 Simulation Example 1:

Consider the nonlinear system referred in Tseng et al. (2001):

$$\dot{x}_1 = Sin\, x_1 - 0.3x_2 + (x_1^2 + 1)u(t)$$
$$\dot{x}_2 = -1.5x_1 - 2x_2 - x_2^3$$
$$y = x_1 \quad (16)$$

A polynomial fuzzy model can be derived by using the sector non-linearity method which accurately describes the system. The obtained polynomial fuzzy system matrices are as follow:

$$A_1(x) = \begin{bmatrix} 1 & -0.3x_2 \\ -1.5 & -2 - x_2^2 \end{bmatrix}$$

$$A_2(x) = \begin{bmatrix} -0.2172 & -0.3x_2 \\ -1.5 & -2 - x_2^2 \end{bmatrix}$$
$$B_1(x) = B_2(x) = \begin{bmatrix} x_1^2 + 1 \\ 0 \end{bmatrix}$$
$$C_1 = C_2 = [1 \quad 0]$$

$$h_1(z) = \frac{Sin\, y + 0.21727}{1.2172y}, h_2(z) = \frac{y - \mathrm{Sin}\, y}{1.2172y}$$

By choosing the reference model as follow:

$$A_r = \begin{bmatrix} -0.5 & 1 \\ -1.5 & -3 \end{bmatrix}, B_r = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_r = [1 \quad 0]$$

and by using the Yalmip toolbox in MATLAB, the problem parameters are obtained:

$$\tilde{P}_{11} = 10^4 \begin{bmatrix} 1.9884 & -0.0004 \\ -0.0007 & 0.0023 \end{bmatrix}$$
$$\tilde{P}_{22} = \begin{bmatrix} 10.08 & 0 \\ 0 & 3.564 \end{bmatrix}$$
$$\tilde{P}_{33} = 10^6 \begin{bmatrix} 1.5134 & 0.4054 \\ 0.4054 & 1.44 \end{bmatrix}$$
$$K_1 = K_2 [-4.5679 \quad 0.0937]$$
$$L_1 = \begin{bmatrix} 69.4129 \\ -1.5001 \end{bmatrix}, L_2 = \begin{bmatrix} 66.0492 \\ -6.1496 \end{bmatrix}$$

The output of the closed loop system in response to the step function input is shown in Fig. 1. As can be seen in Fig. 1, a pulse signal is fed to the reference model and the system output tracks this input well. Fig. 2 and Fig. 3 show the observer estimation of each of the states 1 and 2. It is noticed that the observation operation is well done and although different initial conditions are selected for reference system and the observer in the simulation but after a short time the estimation error of the state is eliminated. Fig. 4 shows the control signal u applied to the system to reduce the tracking error.

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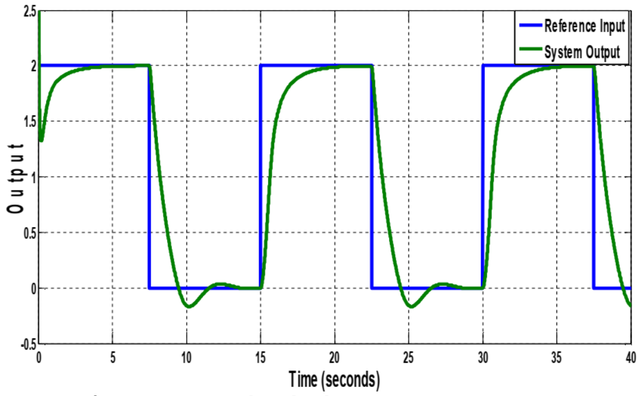

Fig. 1. Reference output tracking by the system.

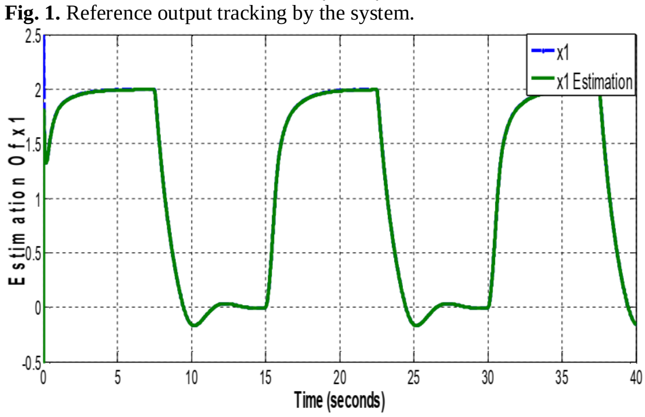

Fig. 2. Estimation of state x_1.

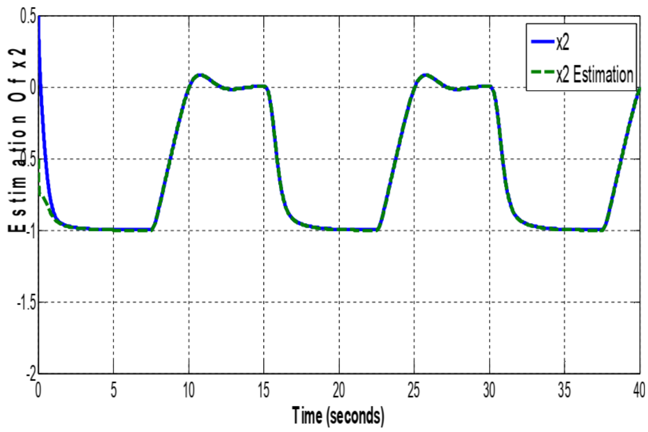

Fig. 3. Estimation of state x_2 .

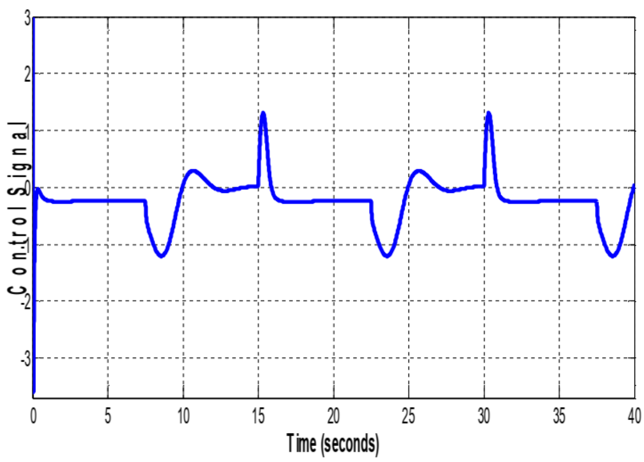

Fig. 4. Control signal.

4.2 Simulation Example 2:

Consider the Lorenz chaotic system in Huang et al. (2014):

$$\begin{cases} \dot{x}_1 = 10(x_2 - x_1) + u(t) \\ \dot{x}_2 = 28x_1 - x_2 - x_1x_3 \\ \dot{x}_3 = x_1x_2 - \frac{8}{3}x_3 \end{cases} \tag{17}$$

for the above system, a TS fuzzy model can be defined as follow:

$$A_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -20 \\ 0 & 20 & -\frac{8}{3} \end{bmatrix}, A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -20 \\ 0 & 20 & -\frac{8}{3} \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C_1 = C_2 = [1 \quad 0 \quad 0]$$

$$h_1(z) = \frac{1}{2}\left(1 + \frac{x_1}{20}\right), h_2(z) = \frac{1}{2}\left(1 - \frac{x_1}{20}\right)$$

Consider the reference model as follow:

$$A_r = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, B_r = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C_r = [1 \quad 0 \quad 0]$$

then with the help of Yalmip Toolbox in MATLAB, we obtain the parameters of the problem.

$$\tilde{P}_{11} = 10^5 \begin{bmatrix} 1.483 & -1.9746 & 0.5047 \\ -1.9746 & 4.0895 & -0.2155 \\ 0.5047 & -0.2155 & 2.2429 \end{bmatrix}$$

$$\tilde{P}_{22} = \begin{bmatrix} 3.3943 & 0.1208 & 0 \\ 0.1208 & 4.0895 & 0 \\ 0 & 0 & 4.0955 \end{bmatrix}$$

$$\tilde{P}_{33} = 10^6 \begin{bmatrix} 2.8669 & 1.6193 & 0.1084 \\ 1.6193 & 7.367 & 0.8463 \\ 0.1084 & 0.8463 & 0.1519 \end{bmatrix}$$

$$K_1 = 10^4[-5.3029 \quad -0.1932 \quad -0.0195]$$

$$K_2 = 10^4[-4.8533 \quad -0.1773 \quad -0.0178]$$

$$L_1 = \begin{bmatrix} 271.1753 \\ 226.9723 \\ -29.5791 \end{bmatrix}, L_2 = \begin{bmatrix} 474.1246 \\ 357.0609 \\ -82.2723 \end{bmatrix}$$

The system simulation results are shown in Fig. 5 to Fig. 9. As you can see in Fig. 5, the output of the reference system tracks the input pulse of the reference model well. Fig. 6, Fig. 7 and Fig. 8 show the fuzzy observer estimation of the states x_1, x_2, x_3, respectively. In order to show the estimation quality, the Lorenz system initial condition [7.5, -9,27] and the observer initial condition [0,0,0] are selected. According to the simulation results from the very first moments, the estimation procedure is performed with high accuracy. Fig. 9 shows the control signal applied to the system.

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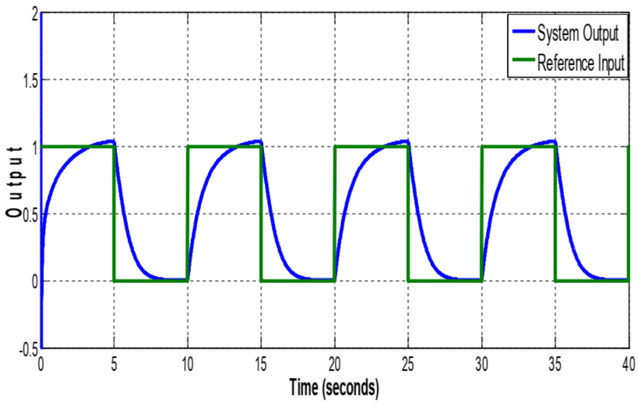

Fig. 5. Reference output tracking by the system.

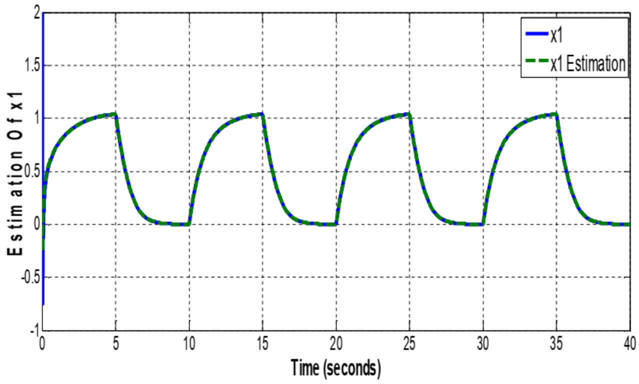

Fig. 6. Estimation of state x_1.

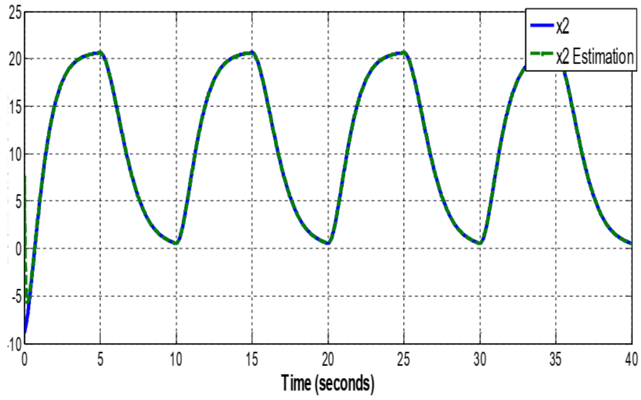

Fig. 7. Estimation of state x_2

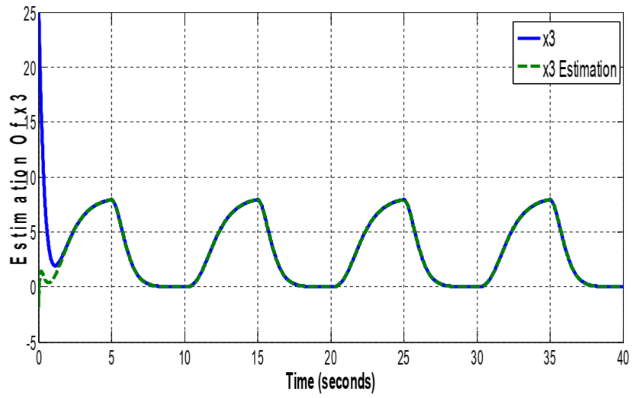

Fig. 8. Estimation of state x_3

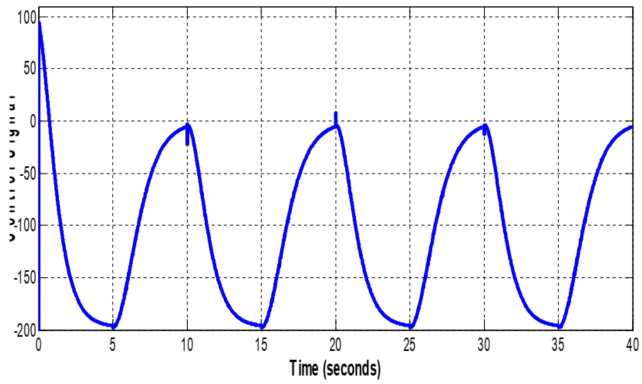

Fig. 9. Control signal

5. Conclusion

This paper is designed for continuous fuzzy polynomial tracking control systems. The design of the tracking control law was based on the design of the polynomial fuzzy observer and the reference model. The conditions presented to guarantee the stability and performance of the closed-loop system are presented based on the infinite norm performance criterion with the help of the sum of squares. As can be seen in the simulation results, this polynomial fuzzy observer shows excellent estimation capability for the system states. Also, the reference input is tracked by the output of the system resulting from the control law designed on the system with high accuracy. Because of the design formulation based on polynomial fuzzy systems, the approach discussed in this paper can be applied to a broader range of nonlinear systems.

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