

## Trade-off Analysis for Multi-Objective Aggregate Production Planning

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### Abstract

Aggregate Production Planning (APP) determines the best way to meet forecast demand in the intermediate future, often from 3 to 18 months ahead, by adjusting regular and overtime production rates, inventory level, labor levels, subcontracting and backorder rates, and other controllable variables. However, the majority of APP models have cost-related objectives, whereas non-cost objectives are often sought by managers. In this study, authors try to minimize total costs and maximize customer service simultaneously. It is shown there is a trade-off between these objectives. Authors propose a linear model for aggregate production planning problem. Then, the two-phase method solution, which takes both objectives into consideration, is used as an alternative objective. By solving the model, it was found that minimizing one objective results in an average loss of about 20% in the other objective. The two-phase method solution, on the other hand, results in a loss of 8% from the furthest objective and 7% from the closest objective.

Keywords: Aggregate production planning, Customer service, Trade-off, Two-phase method

### 1. Introduction

Aggregate Production Planning (APP) determines the best way to meet forecast demand in the intermediate future, often from 3 to 18 months ahead, by adjusting regular and overtime production rates, inventory level, labor levels, subcontracting and backorder rates, and other controllable variables (Mortezaei et al., 2013). APP aims to identify production, inventory and work force levels to meet fluctuating demand requirements over an intermediate-range planning horizon. Generally, APP takes either one or mix of several pure strategies in giving a respond to fluctuating market demand: 1- adjusting production rate 2- adjusting workforce or subcontracting 3- maintaining a constant production level with inventory. Saad (1990) categorizes all models for solving APP problems into six categories 1- linear programming 2- linear decision rule 3- transportation method 4- management coefficient rule 5- search decision rule 6- simulation. In this section, we review some previous researches. Fung et al. (2003) investigated a fuzzy multiple products APP problem in their research. Only one objective function (minimizing the total cost) is considered in their fuzzy multi-product APP model and he defined the demands and the capacities as triangular fuzzy numbers. The problem is transformed into a crisp parametric programming problem with using the membership functions of the fuzzy parameters. The resultant crisp problem is solved by utilizing parametric programming and the proposed interactive method. They employed threshold level to state the decision maker's

satisfaction level with the usage of production capacity which should not be less than threshold level. Techawiboonwong and Yenradee (2003) developed a linear model for multi-product APP that enabled workers to be transferred among production lines. Their model had only one objective function or minimizing total cost. In reality, when a worker performs any task for a long time, they get used to the task. Then, if that worker is transferred to operate a different task, their skill with the new task would likely be lower than that with the old task. This also impacts productivity. A feasible way to evaluate the extent to which productivity falls is to evaluate training cost and cost due to the loss of production. Therefore, they add to their model cost of transferring workers as a cost parameter. They compared two situations: 1) the worker cannot transfer among production lines, and 2) the worker can transfer between lines. The results of APP model showed that the total cost increases about 5% when the workers did not transfer among production lines; however, the goals and model inputs in these APP models were assumed to be crisp. Recently, Jamalnia and Feili (2013), Li et al. (2013), Ning et al. (2013), Mortezaei and Zulkifli (2013, 2014), Mortezaei et al. (2014) and Iris and Cevikcan (2014) studied on aggregate production planning. However, the majority of APP models have cost-related objectives, whereas non-cost objectives are often sought by managers. This paper aims to consider total costs and customer service as performance measures and authors tries to find out trade-off between them.

## 2. Aggregate production planning model

This model wants to find, for each product and each planning period, the 'best' production level and inventory level. It also wants to find the backorder level for each period. The model assumptions are as follows.

1. Production system cannot operate with less than a certain number of workers (minimum workforce level);
2. All model parameters are deterministic;
3. Raw materials are always available without shortage;
4. Average hiring and firing cost are considered;
5. Overtime and subcontracting are not allowed.

The two objective multi-period multi-product APP model can be formulated as follows:

Indices:

N: product type

T: planning period

Decision variables:

$p_{nt}$ : units of production for product n in period t (units)

$I_{nt}$ : inventory level for product n in period t (units)

$B_{nt}$ : backorder level for product n in period t (units)

Parameters and constants:

N: total number of products

T: total number of planning periods in the planning horizon

$cp_{nt}$ : production cost for product n in period t (\$/unit)

$ci_{nt}$ : inventory carrying cost for product n in period t (\$/unit-period)

$H_n$ : hours of labor needed for each unit for product n

$U_{nt}$ : hours of machine usage per unit of nth product in period t (machine-hour/unit)

$R_t$ : maximum working hours in period t (a month), equal to regular working hours per worker per day × working shift in per day × working days for period t

$I_{n0}$ : initial inventory level for product n (units)

$B_{n0}$ : initial backorder level for product n (units)

$D_{nt}$ : forecasted demand for product n in period t (units)

$D_{min}$ : minimum demand for product n in period t (units)

$M_{t \max}$ : maximum machine capacity available in period t (machine-hour)

Minimize total cost:

$$\text{Min } Z_1 = \sum_{n=1}^N \sum_{t=1}^T cp_t \times p_{nt} + I_{nt} \times ci_{nt} \quad (1)$$

Maximize customer service:

$$\text{Max } Z_2 = (1 - \sum_{n=1}^N \sum_{t=1}^T B_{nt} / \sum_{n=1}^N \sum_{t=1}^T D_{nt}) \quad (2)$$

$$\sum_{n=1}^N H_n \times p_{nt} \leq R_t \times w_t \quad t = 1, \dots, T \quad (3)$$

$$p_{nt} - I_{nt} + B_{nt} + I_{n,t-1} - B_{n,t-1} = D_{nt} \quad (4)$$

$$n = 1, \dots, N, t = 1, \dots, T$$

$$p_{nt} + I_{n,t-1} - B_{n,t-1} \geq D_{min,n} \quad (5)$$

$$n = 1, \dots, N, t = 1, \dots, T$$

$$\sum_{n=1}^N U_{nt} \times p_{nt} \leq M_{t \max} \quad t = 1, \dots, T \quad (6)$$

$$p_{nt}, I_{nt}, B_{nt} \geq 0; \quad (7)$$

$$n = 1, \dots, N; t = 1, \dots, T$$

## 3. Methods for solving multi-objective models

Zimmermann (1978) proposed the first fuzzy approach for solving multi-objective linear programming problems, called max–min approach. Max-min is single-phase method which tends to maximize overall satisfaction degree of objective. Another method for solving multi-objective problem is two phase method which suggested by Chen and Chou (1996). In two-phase method also called lexicographic method, problem decompose two-level sub problem and then solve iteratively. In the other word, first we apply max-min method for computing total satisfaction degree ( $\lambda$ ) then based on this  $\lambda$  we solve following equation.

$$\text{Max } \bar{\lambda} = 1/(1+r) \sum_{i=1}^{l+r} \lambda_i \quad (8)$$

Subject to:

$$\lambda \leq \lambda_k \leq (z_k(x) - z_k^{NIS}) / (z_k^{PIS} - z_k^{NIS}), \quad k=1, \dots, L$$

$$\lambda \leq \lambda_{s+1} \leq (w_s^{NIS} - w_s(x)) / (w_s^{NIS} - w_s^{PIS}),$$

$$s=1, \dots, r,$$

$$\lambda_k, \lambda_s \in [0,1] \tag{9}$$

$$x \in X.$$

**4. Computational study**

The trade-off between the total costs, and customer service are investigated in this section, by considering following questions:

1. When minimizing single objective (total costs or customer) what is the loss in the non-optimized objective? For answering this question three following performance measures will be used.

$$Total\ costs\ (range) = \frac{(total\ cost^{cs} - total\ costs^*)}{total\ costs^*} \tag{10}$$

Where total cost<sup>cs</sup> is the total costs value obtained for optimal customer service solution.

$$customer\ service\ (range) = \frac{(customer\ service^{TC} - customer\ service^*)}{customer\ service^*} \tag{11}$$

Where customer service<sup>TC</sup> is the customer service value obtained for optimal total costs solution.

$$MaxMax = \frac{Max(Total\ costs\ (range),\ customer\ service\ (range))}{customer\ service^*} \tag{12}$$

2. What is the quality of the max-min solution (how close is the Two-phase solution to the optimal objectives' values)? For answering this question two following performance measures will be used.

$$MinMax = \frac{\max\left(\frac{(total\ cost^{mx} - total\ costs^*)}{total\ costs^*}, \frac{(customer\ service^* - customer\ service^{mx})}{customer\ service^*}\right)}{customer\ service^*} \tag{13}$$

Where total cost<sup>mx</sup> and customer service<sup>mx</sup> are the optimal two-phase solutions.

$$MinMin = \frac{\min\left(\frac{(total\ cost^{mx} - total\ costs^*)}{total\ costs^*}, \frac{(customer\ service^* - customer\ service^{mx})}{customer\ service^*}\right)}{customer\ service^*} \tag{14}$$

Following randomly generated example will be solved. There are three products in three planning periods. Production costs are 4, 5, and 7 for products 1 to 3 respectively and inventory carrying costs are 1, 1, and 2 for products 1 to 3 respectively. Minimum demand for products are 50 and the hours of labor needed for each unit production, Hours of machine usage, Maximum machine capacities and Maximum working hours are 0.013, 0.0002, 744 and 600 respectively. Initial inventory and backorder are zero. Demands are 70, 90, and 110 for products 1 to 3 respectively. Table 1 gives results of this example.

**Table 1** Results of example.

product	period	Total costs (as single objective)			Customer service (as single objective)			Two-phase method for this bi-objective model		
		Production quantity	inventory	backorder	Production quantity	inventory	backorder	Production quantity	inventory	backorder
1	1	70	0	0	70	0	0	70	0	0
1	2	50	0	20	70	0	0	70	0	0
1	3	70	0	20	70	0	0	70	0	0
2	1	90	0	0	90	0	0	90	0	0
2	2	50	0	40	90	0	0	90	0	0
2	3	90	0	40	90	0	0	90	0	0
3	1	110	0	0	110	0	0	110	0	0
3	2	110	0	0	110	0	0	110	0	0
3	3	50	0	60	110	0	0	50	0	60
Objective values		3800			1 (or 100%)			Total costs=4088		Customer service=0.92

## 5. Conclusion

In this study, authors tried to minimize total costs and maximize customer service simultaneously. It was shown there was a trade-off between these objectives. We proposed a linear model for aggregate production planning. Then, the two-phase method solution, which takes both objectives into consideration, was used as an alternative objective. By solving the model, it was found that minimizing one objective results in an average loss of about 20% in the other objective. The two-phase method solution, on the other hand, results in a loss of 8% from the furthest objective and 7% from the closest objective. For further studies, performing a design of experiments including different factors such as the problem size and demand is suggested to analyse effect of each factor on the objectives.

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