

## **Enhancement of the Satellite Attitude Control Using Optimal Regulator Scheme**

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### **Abstract**

Accuracy of satellite control for space investigation has priority during its mission. This study aims to design a controller for the satellite attitude control which carries scientific sensor package. The modeling of the satellite with its electronic device is carried out in order to capture the states of systems then the appropriate controller strategy is applied to keep the satellite at the fixed value. In addition, to satisfy the design characteristics two control schemes in comparative manner are discussed namely, state feed-back controller and linear quadratic regulator (LQR). The results show LQR helps satellite to follow its reference signal along with desired overshoot and settling time.

Keywords: Satellite Attitude Control, Optimal Regulator, Settling time, Controller

### **1. Introduction**

Exploration for perceiving tremendous phenomenon in the space has been the one of the interminable desire for human being in throughout the history of the human life. These days by improving the technology the novel devices have been sent into the space for universe investigation purposes. Among of various equipments, satellites have been used prevalently. Satellites are commonly utilized as data and information conveyer between earth and surrounding space. Therefore, control of satellite in the space come into picture as essential aspect for cosmos investigation. As matter of fact in designing of satellite, position an orientation control of satellite in the space is major consideration for designer (Zhang et al., 2013; Ahmed and Kerrigan, 2014). It can be said that receiving or transmitting accurate signals depends on how well satellite placed in the space.

Keeping the satellite in the desirable attitude is one of the real obsessions of scientist and control engineers because of existing of disturbances such as solar pressure (Bai and Wu, 2014), micrometeorites, electrical noises and other floating cosmic objects causing deviation of the satellite attitude from normal path or its direction (Ovchinnikov and Ivanov, 2014). Moreover, most of satellites are carrying the electro-communication devises thus, fluctuation in attitude result in missing the signal from transmitter. Of course, attitude stabilization during the satellite mission should be taking into account by control engineers (Haichao et al., 2013; Di et al., 2014). It follows that; to rely on transmitted information the package must be isolated against any noise and disturbance to stay in desired position. This sustained position can relatively be assigned

by pointing either one star or group of stars through the limitless space. The maneuverability and attitude are controlled by star tracker method (Yuwang et al., 2014).

One of the current methods to track and sustain satellite in the specific position is using the light of stars in space to keep the relative position of satellite with respect to shiny star which called star tracker control (Birbaum, 1996; Kai et al., 2013; Samaan and Theil, 2012). In Kai et al., (2013) it was shown the autonomous navigation by implementing a relative position measurement between a group of satellites using star sensors and inter-satellite links. They applied the navigation method which is based on dynamic and measurement model. Some researcher focus on improving sensors in term of signal processing technique in which novel method is used to reduce star tracker navigation error such as measurement uncertainties and increase the controllability of satellite (Rufino and Accardo, 2002).

As reported by Ho (2012) identification methods are uniquely used to track lost stars in spaces that also called star identification. In this study two phases are discussed, firstly feature extraction and secondly for catalogue search. As it said earlier attitude control of satellite is presented in large amount of research and also case studies. For example; Wang et al, (2013) applied regularized robust filter for attitude control system for star trackers in which relative installation error of star tracker in attitude measurement data is controlled. To eliminate the low-frequency periodic error of star tracker the kalman filter method is utilized in which this error can be identified by extracting the Fourier parameters of estimated gyro drift (Jiongqi et al., 2012). Attitude estimation algorithm for the satellite with three star trackers is implemented by Chen et al. (2012) in which the extended kalman filter is used to

estimate the attitude of the nonlinear satellite system. The modeling part is carried out by quaternion to describe nonsingular attitude description. Except attitude control, orbit control is also investigated by Canuto (2013) in which both case are dealing with separately. Dynamic of orbit movement and formation is obtained from perturbation formulation for designing a hierarchical control design in order to delete non-gravitational and angular acceleration.

The attitude control of satellites is a significant objective of researchers during past decade of studies. Based on conventional methods modeling of satellite is carried out with various approaches depending on design specification and different uncontrollable external factors. As further investigation in attitude control of satellite this study aim to design a satellite's attitude control the geosynchronous communication satellite IPSTAR as case study which addressed in (Franklin et al., 2002 ). The proposed satellite is composed of main body as carrier, and the scientific sensor package to capture and transmit the cosmic events. The main aim of this study is to present the appropriate and satisfactory controller strategy in order to meet the best satellite's attitude in terms of desired dynamic response in the space, especially when the satellite is moving to settle in the new position. Furthermore; the comparative control approach has been intended in this study namely, state-feedback controller and linear quadratic regulator (LQR).

2. Modeling

In order to place the sensor package device into correct direction, the star tracker technique is used. In this method the images of certain star are recorded by star tracker's focal plane (Yuwang et al., 2014). For sake of simplicity in modeling part, only maneuverability of satellite is concerned and discussion on star tracker measurement and corresponding model for attitude determination is left and well discussed in other studies (Yuwang et al., 2014), thus the association of actuator and sensor modeling with main satellite body is neglected to solely illustrate the primitive modeling and control of satellite attitude control.

The Lumped Parameter model is employed for driving of the governing equation of satellite and scientific device. In Fig.1, the configuration of satellite and attached sensor are illustrated in which main body of satellite and attached sensor considered as two lumped masses with corresponding rotational displacement. However, the effect of solar pressure, orbit perturbation and micrometeorites are neglected. The flexibility between satellite body and sensor is assigned as spring and damper with constant value and coefficient respectively. The sufficient external force for driving and moving the satellite in the specific attitude could be supplied by gas jet thruster and magnetic torques which is considered in this study. For driving the mathematical model of satellite the simplified model is used, see Fig.2.

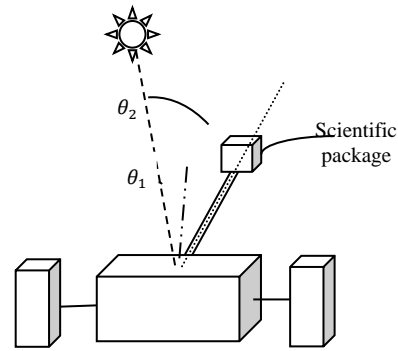


Fig. 1. Schematic of geosynchronous communication satellite

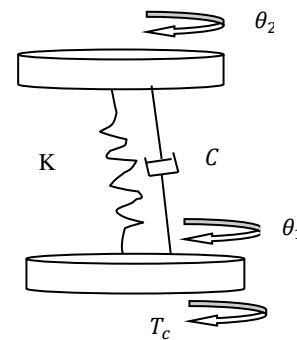


Fig. 2. Simplified model of geosynchronous communication satellite

2.1 State space model

Any dynamic system can be described by state-space model as:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = cx \end{cases} \quad (1)$$

where  $x$  is the system states,  $A$  is system matrix,  $B$  is input matrix and  $c$  is output matrix.

The linear state- space model with unknown parameters is driven by applying the newton second law over lumped model as illustrated in the Fig.2. The desired coordination should be kept by  $\theta_2$  by which the satellite adjusts its position with shiny star that is may located billion light years far from our satellite and its sensor. The controller strategy must satisfy the exact value of the  $\theta_2$  in terms of scientific investigation. The ordinary differential equation of two lumped mass with stiffness and damper is described as

$$\begin{aligned} J_1 \ddot{\theta}_1 &= -b(\dot{\theta}_1 - \dot{\theta}_2) - k(\theta_1 - \theta_2) + T_c \\ J_2 \ddot{\theta}_2 &= b(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) \end{aligned} \quad (2)$$

where  $\theta_1$  is the position of main body of satellite,  $T_c$  is the controlling force imposed on main body,  $J_1$  is the inertia of the main body,  $J_2$  is the inertia of the instrument package and  $k$  and  $b$  is the stiffness and damping of links between main body and instrument package respectively.

$$\begin{cases} J_1 \ddot{\theta}_1 + b\dot{\theta}_1 - b\dot{\theta}_2 + k\theta_1 - k\theta_2 = T_c \\ J_2 \ddot{\theta}_2 + b\dot{\theta}_2 - b\dot{\theta}_1 - k\theta_2 - k\theta_1 = 0 \end{cases} \quad (3)$$

$$\begin{aligned} \ddot{\theta}_1 = \frac{d^2\theta_1}{dt^2} = \frac{dv_1}{dt} = \dot{v}_1 \text{ or } \frac{d\theta_1}{dt} = v_1 \\ = \dot{\theta}_1 \end{aligned} \quad (4)$$

$$\ddot{\theta}_2 = \frac{d^2\theta_2}{dt^2} = \frac{dv_2}{dt} = \dot{v}_2 \text{ or } \frac{d\theta_2}{dt} = v_2 = \dot{\theta}_2$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{v}_1 \\ \dot{\theta}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/J_1 & -b/J_1 & k/J_1 & b/J_1 \\ 0 & 0 & 0 & 1 \\ k/J_2 & b/J_2 & -k/J_2 & -b/J_2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ v_1 \\ \theta_2 \\ v_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} T_c$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ v_1 \\ \theta_2 \\ v_2 \end{bmatrix} + 0 \quad (5)$$

By implementing the experimental test on real prototype (Franklin et al., 2002) the value for K, b, J<sub>1</sub> and J<sub>2</sub> are defined as follow,

$$0.09 \leq k \leq 0.4, \quad 0.038\sqrt{k/10} \leq b \leq$$

$$0.2\sqrt{k/10}, \quad k = 0.0036, \quad J_1 = 1, \quad J_2 = 0.1$$

By substituting those parameters into Eq. (5) the final stste-space equation of satellite can be represented as,

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{v}_1 \\ \dot{\theta}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.091 & -0.0036 & 0.091 & 0.0336 \\ 0 & 0 & 0 & 1 \\ 0.91 & 0.036 & -0.91 & -0.036 \end{bmatrix} \begin{bmatrix} \theta_1 \\ v_1 \\ \theta_2 \\ v_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} T_c$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ v_1 \\ \theta_2 \\ v_2 \end{bmatrix} + 0 \quad (6)$$

Moreover, the corresponding transfer function of the state space model can be found by:

$$G(s) = c(sI - A)^{-1}B \quad (7)$$

where I is identity matrix. By substituting known parameters into Eq. (7) the system transfer function can be written as:

$$G(s) = \frac{0.036(s + 25)}{s^2(s^2 + 0.04s + 1)} \quad (8)$$

### 2.1.1 Stability characteristic

As mentioned earlier, the state-feedback controller is the control strategy to keep the satellite into fixed position. As rule of thumb, state-feedback controller introduced new poles for close loop system. Before finding desired poles in order to reach design specification, first of all the location of poles of uncontrolled system (open loop) should be obtained. The root-locus technique is employed to show the location of the poles, see Fig.3.

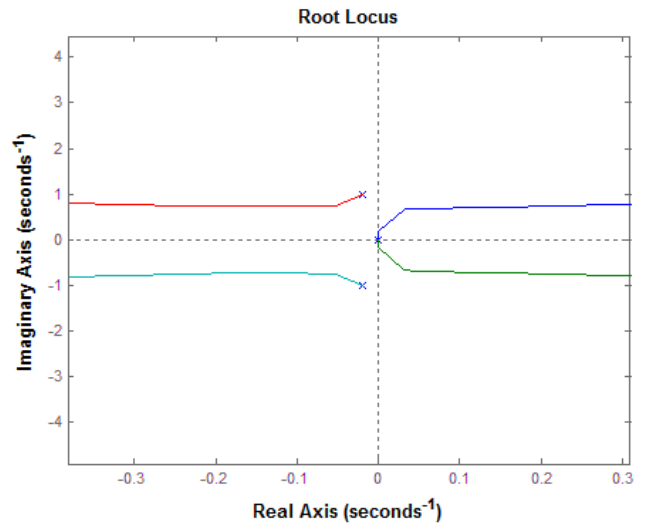


Fig. 3. Location of poles of uncontrolled satellite system

Complex plan indicates there are two poles which make system unstable by locating in the right hand side of complex plane (RHP). Pole-placemat controller tries to place the poles of final close loop system into left hand side complex plane (LHP) by which the system become stable and satisfy the design requirements as well.

### 3. Controller design

In this section the state feedback controller is proposed in order to stabilize system and attitude control of the satellite. The prime strategy of state-feedback is modifying and creating new close loop system by moving the unstable poles of transfer function from right hand side to left hand side in the complex plane. In the state feedback controller the characteristic equation of open loop system manipulated in order to find desired gain for close-loop system. By introducing a new input to satisfy design characteristic as:

$$u = k_0 r - kx \quad (9)$$

where k and k<sub>0</sub> are controller input gains where k<sub>0</sub> is neglected because it does not affect on transient response.

The characteristic equation of open-loop system expressed as:

$$|SI - A| = 0 \tag{10}$$

By substituting the known parameters into Eq. (10) it can be written as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.091 & -0.0036 & 0.091 & 0.0336 \\ 0 & 0 & 0 & 1 \\ 0.91 & 0.036 & -0.91 & -0.036 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

The characteristic equation is unstable, thus in order to stabilized system, the new input should be added into the system as:

$$|SI - A + Bk| = 0 \tag{12}$$

where  $k$  is a constant matrix which also considered as close-loop system gains by which the system is stable and satisfy control design characteristics as well.

The block diagram of state feed-back controller is depicted in Fig.4.

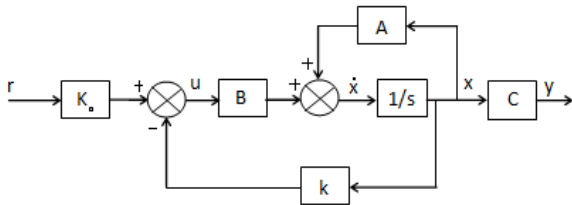


Fig. 4. Block diagram of state feedback controller

By substituting the known parameters into Eq. (12) the parameters of matrix  $k$  can be found as:

$$\begin{vmatrix} s & 1 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{vmatrix} - \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.091 & -0.0036 & 0.091 & 0.0336 \\ 0 & 0 & 0 & 1 \\ 0.91 & 0.036 & -0.91 & -0.036 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} [K_1 \ K_2 \ K_3 \ K_4] = 0 \tag{13}$$

The determinant of Eq. (13) can be written as fourth order polynomial as follow:

$$s^4 + (k_2 + 0.04)s^3 + (k_1 + 0.036k_2 + 0.036k_4 + 0.99)s^2 + \dots$$

$$\dots (-0.03 + 0.03k_1 + 0.91k_2 + 0.036k_3 + 0.91k_4)s + 0.91k_1 + 0.91k_3 = 0 \tag{14}$$

For finding the value of  $k_1, \dots, k_4$ , Eq. (14) should be equivalent to desired characteristic equation which defined from design characteristics.

### 3.1 Design characteristic

The desired characteristic equation would be defined by considering the design specifications. By referring to the (Franklin et al., 2002), the design criteria of satellite are 15% overshoot and 20 second settle time during the attitude control. By considering these two values the dominant poles can be obtained by introducing a new characteristic equation in which poles are lay down in the LHP.

$$G(s) = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{15}$$

The unknown parameters of Eq. (15) are defined using design specifications as:

$$Mp = \frac{e^{-\xi\pi/\sqrt{1-\xi^2}}}{\xi} \tag{16}$$

where  $Mp$  is the overshoot of system and  $\xi$  is the damping ratio.

$$T_s = \frac{4}{\xi\omega_n} \tag{17}$$

where  $T_s$  is the setting time of system and  $\omega_n$  is the natural frequency of system; thus, from Eq. (16) and (17) it can be concluded that:  $\xi = 0.5$   $\omega_n = 0.38$ .

By replacing the value of  $\xi$  and  $\omega_n$  into Eq. (15) the new characteristic equation can be as:

$$G(s) = \frac{0.145}{(s^2 + 0.4s + 0.156)} \tag{18}$$

The Eq. (18) presents stable open loop system which all poles located in the LHP. Fig 5 depicts the position of the new poles.

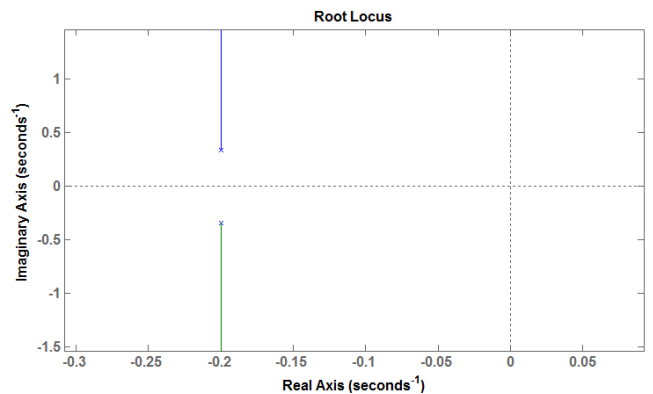


Fig. 5. Location of poles of stable system

Along with two desired poles, another two arbitrary poles selected to make the fourth degree polynomial same as initial transfer function of the system. Therefore, the final transfer function of system can be written as:

$$G(s) = \frac{0.036(s + 25)}{(s + 1.45)(s + 4)(s^2 + 0.4s + 0.156)} \tag{19}$$

The step response of new transfer function which fulfills all design consideration illustrated in Fig.6.

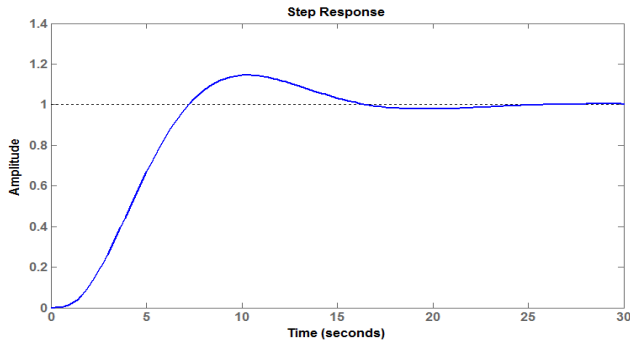


Fig. 6. Step response of new transfer function

To find the close-loop system gains – matrix  $k$  - the characteristic equation of Eq. (18) must be equal to Eq. (14). The result of this comparison presents matrix:

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 5.8 \\ -6.02 \\ -2.3 \end{bmatrix} \quad (20)$$

3.2. State-feedback controller implementation

In this section the state-feedback controller is implemented on system to control an attitude of the satellite. The main function of this controller is pivoted on matrix  $k$  that provides the gains of close loop system. As shown in Fig. 7 the gains are located at the feedback of the close loop system to tune response of system as desired response. the step response of the close-loop system indicates that the satellite attitude follow desired response or design characteristics. In this case the desired out put is the attitude of instrument sensor which is assigned as  $\theta_2$  in Eq. (4). The corresponding gain to adjust the value of  $\theta_2$  is  $k_3$  that found in Eq. (20). The step response of the system, see Fig.8, shows the selected gain ( $k_3$ ) fulfil the overshoot percentage of design specification; however, the settling time is not as desirable as we expect. The closed loop system is settle down after 50 second.

3.3. Linear quadratic feedback controller

As it is shown the state-feedback controller is not sufficient approach to satisfy the design specification in terms of settling time value. Thus, Linear Quadratic Regulator (LQR) as another alternative controller is used to overcome the settling time problem.

LQR controller is optimal controller scheme in which the gains are selected to minimize the performance index as:

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \quad (21)$$

where  $x$  is state matrix,  $Q$  is positive semi-definite matrix which influence on performance of controller by tuning of controller output ,  $R$  is positive definite matrix which defines the power of controller output and  $u$  is system input.

The optimal gains of closed loop system are defined by:

$$k = R^{-1} B^T p(t) \quad (22)$$

where  $p(t)$  is the solution of the Riicati equation in form of:

$$A^T p(t) + p A - p(t) B R^{-1} B^T p(t) = 0 \quad (23)$$

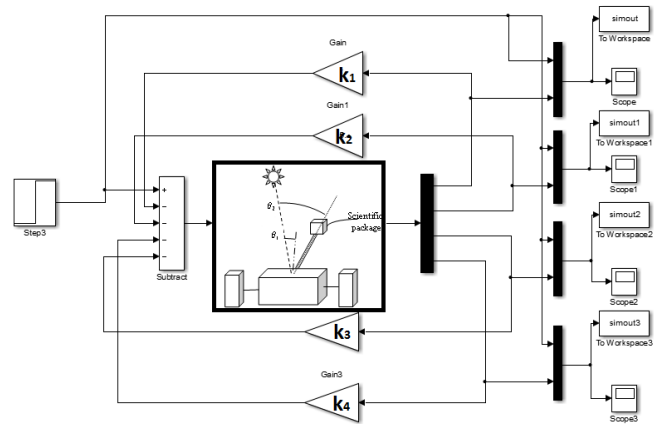


Fig. 7. Implementation of state feed-back controller on satellite

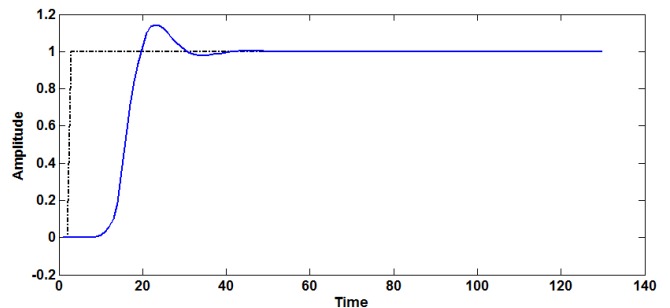


Fig. 8. Step response of close- loop system

By solving the solution of Eq. (22) and Eq. (23) the optimal input -Eq. (9) - which minimize the controller output and the allocated energy to reach to the design characteristics as well. The step response of the closed loop system including LQR controller is illustrated in Fig.9.

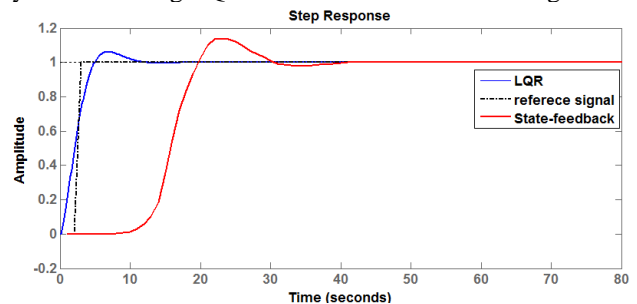


Fig. 9. Step response of close- loop system using LQR and state-feedback controller

As illustrated in Fig.9, the LQR controller has ability to satisfy the design characteristics by dedicating less overshoot and short settling time (12 second) compare to the state-feedback controller. The result of bode diagram in Fig.10 depicted that LQR elevates the system speed with higher band width than state-feedback controller.

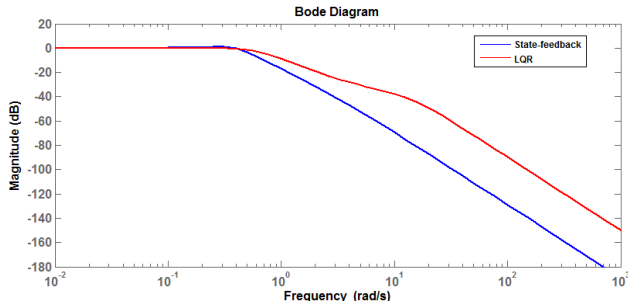


Fig. 10. Bode diagram of the LQR and State-feedback controller

#### 4. Conclusion

The attitude control of satellite is carried out by two different control schemes in which state-feedback controller and LQR applied on state space model of satellite. Both controllers helps satellite to follow reference signal; however, LQR presents significant contribution than state-feedback controller. The LQR provides design characteristics by dedicating less overshoot and short settling time. The next step of study will consider the effect of outer disturbances on system and try to employed robust controller for satellite attitude control.

#### References

- Bai, X., & Wu, X. (2014). 1-Bit processing based model predictive control for fractionated satellite missions. *Acta Astronautica*, 95.
- Di, B., Min, J., & Zhao, Y. (2014). Nonlinear analysis and vibration suppression control for a rigid – flexible coupling satellite antenna system composed of laminated shell reflector. *Acta Astronautica*, 96, 269–279.
- Haichao, G., Lei, J., & Shijie, X. (2013). Local controllability and stabilization of spacecraft attitude by two single-gimbal control moment gyros. *Chinese Journal of Aeronautics*, 26(5), 1218–1226.
- Ovchinnikov, M., & Ivanov, D. (2014). *Acta Astronautica* Approach to study satellite attitude determination algorithms. *Acta Astronautica*, 98,133–137.
- Zhang, B., Liu, K., & Xiang, J. (2013). A stabilized optimal nonlinear feedback control for satellite attitude tracking. *Aerospace Science and Technology*, 27(1), 17–24.
- G.F.Franklin. (1988). *Digital Control Of Dynamic Systems*, 2nd ed., Prentice Hall, Upper Sadedle River,Nj.
- Ahmed, S., & Kerrigan, E. (2014). Suboptimal predictive control for satellite detumbling. *Journal of Guidance Control, and Dynamics*, 37(3),850- 859.
- Birnbaum, M. (1996). Spacecraft attitude control using star filed trackers. *Acta Astronautica* , 39(9-12), 763-773.
- Kai, X., Chunling, W., & Liangdong, L. (2013). Autonomous navigation for a group of satellites with star sensors and inter-satellite links. *Acta Astronautica*, 86, 10-23.
- Rufino, G., Accardo, D. (2002). Enhancemet of the centroiding algorithm for star tracker measure refinement. *Acta Astronautica*, 53, 135-147.
- Ho, k. (2012). A survey of algorithms for star identification with low-cost star trackers. *Acta Astronautica*. 73, 156-163.
- Wang, J., H, Z,M., Zhou, H,Y.,& Jiao, Y,Y. ( 2013). Regularized robust filter for attitude determination system with relative installation error of star trackers. *Acta Astronautica*, 87, 88-95.
- Yuwang, L., Junhong, L., Yonghe D., Defeng G., & Dongyun, Y. (2014). Precession–nutaton correction for star tracker attitude measurement of STECE satellite.*Chinese journal of Aeronautics*, 27(1), 117-123.
- Jiongqi, W., Kai, X., & Haiyin, Z. (2012). Low frquency error identification and compensation for star tracker attitude measurement. *Chinese Journal of Aeronautics*, 25, 615-621.
- Samaan, M., & Theil, S. (2012). Development of a low cost star tracker for the SHEFEX mission. *Aerospace science and technology*, 23, 469-478.
- Chen, B., Geng, Y., & Yang, X. (2012). High precision attitude estimation algorithm using three star trackers. *Proceeding of the 10th world congress on intelligent control and automation*, july 6-8, Beijing, China.
- Canuto, E. (2013). Orbit and attitude control for gravimetry drag-free satellites:when disturbance rejection becomes mandatory. *American conference (ACC) washangton, DC,USA*, June, 17-19.